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Fast and Stable Surface Feature Simulation for Particle-Based Fluids

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Abstract: In order to efficiently and realistically capture microscopic features of fluid surface, a fast and stable surface feature simulation approach for particle-based fluids is presented in this paper. This method employs a steady tension and adhesion model to construct surface features with the consideration of the adsorption effect of fluid to solid. Molecular cohesion and surface area minimization are appended for surface tension, and adhesion is added to better show the microscopic characteristics of fluid surface. Besides, the model is integrated to an implicit incompressible smoothed particle hydrodynamics (SPH) method to improve the efficiency and stability of simulation. The experimental results demonstrate that the method can better simulates surface features in a variety of scenarios stably and efficiently.

Key words: virtual reality; 3D visualization; fluid simulation; surface feature

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With the increasing demands for details and realistic effects of fluid simulation, the modeling of surface feature is becoming more and more necessary. Surface tension is one of the most common and important physical characteristics to reveal surface features of fluids, which is generated by molecules cohesion. Because the density of particles at the fluid-air interface is underestimated due to lacking neighbor particles, which results in negative pressure and particle clustering. Besides, the surface feature simulation is usually time-consuming, which has the problem of time step restriction and numerical instability. Therefore, using a particle-based method, for instance, smoothed particle hydrodynamics (SPH), simulates surface feature realistically is still a challenging topic. To solve these problems, we propose a surface feature simulation method for particle-based fluids to improve the computational efficiency and stability with the good surface tension and adsorption effect.

Nowadays, surface tension models are mainly classified into two categories: macro model and micro model. The former is also known as continuous surface force model (CSF)[1]. It is a kind of method based on color field, the value of which jumps at two phase interface[1-4]. These methods usually first interpolate particles' color field using SPH formula. Then, the normal vector of surface is calculated according to color field gradient. Finally, surface curvature and surface tension could be constructed. But deviations always exist in calculating the surface's normal vector, especially where the surface has sharp corners and less particles. This leads into a large computational error in surface curvature mean while creates an asymmetric force for fluid particles which could not ensure momentum conservation.

The micro models construct surface tension by particles' cohesion which is imitating molecu-

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lar attraction[5-7]. Compared to the macro model, the micro model avoids the calculation of color field's second derivative and surface curvature which are sensitive to particles' derangement. Moreover, micro model is simple to implement and efficient. Nugent et al.[5] used van der Waals equation of state to calculate the attraction pressure, and expanded the attraction range to obtain a stable droplet effect. Tartakovsky et al.[7] presented a molecular cohesion model to produce surface tension for numerical simulation. They control particles' attraction and repulsion by a cosine function. Becker et al.[6] employed SPH kernel function instead of cosine function, making the calculation of attraction in a specified range. In fact, both the cohesion models have the traits of repulsive force in short-range and attractive force in long-range.

However, micro and macro models cannot simulate both large surface tension and the surface minimum curvature effect well at the same time, and may cause problems such as particle clustering and momentum violation. Akinci et al.[8] dealt with the problems through the cohesion-repulsion model, and added the adsorption force model. Inspired by this, we proposed an improved surface tension and adsorption force model and combined it with the implicit incompressible SPH method (IISPH), which can simulate the fluid surface microscopic features better and enhance the numerical stability and computational efficiency.

1 Related Work

Since the surface tension has important effects on the details of the fluid simulation, many studies have been conducted to suit for fluid modeling. In the particle-based fluid simulation category, previous research minimized the surface curvature to achieve the surface tension effect[2-3]. However, when using these methods, surface curvature is very sensitive to particle disorder, the external force is asymmetrical and does not keep the momentum conservation. To eliminate these problems, researchers used adjacent fluid particles cohesion at the molecular level to improve the early surface tension models[6-7]. Nevertheless, only using cohesion cannot implement surface area minimization and large surface tension would cause unreal flow. Clavet et al.[9] produced surface tension by the gravitational force. The proposed double density relaxation model can produce a better surface tension effect, but it isn't suitable for simulating low viscous liquids. Yu et al.[10] proposed a SPH-based surface tension method in which the curvature is estimated on the surface mesh and surface tension is applied to the adjacent fluid particles surrounded by the grid. But its surface tracking method may not be able to detect isolated fluid blocks so that the surface tension effect cannot be exhibited in these areas, which also depends on the resolution of the tracking grid. Akinci et al.[8] created surface tension by constructing inter-particle interactions, handling large surface tension while maintaining momentum conservation, and repelling repulsive particles at too close distances, prevented the free surface of the particle cluster problem without applying additional operation.

The adsorption force reflects the effects of fluids attracted by other substances, which is also a way to show surface feature. Steele et al.[11] proposed a Lagrangian simulation method for viscous fluids that achieves the adsorption effect. They used distance-dependent forces to define the adsorption properties of different types of substances, but the using of linear density cores with strict anti-penetration constraints limits the ability to simulate highly viscous fluids. Then, Clavet et al.[9] simulated the viscoelastic SPH fluid's adhesion through a distance-based attraction term. Schechter and Bridson[12] realized adhesion by calculating ghost velocity at solid particles that using the velocity of the solid itself and the tangential component of the velocity of the nearest fluid particle. In the method of He et al.[13], the adhesion effect is achieved through the edge effect of different slip conditions created by velocity constraints.

Akinci et al.[8] realized a physically reasonable fluid-solid adhesion without additional treatments (e. g. ghost SPH), while ensured the momentum conservation.

2 Fluid Simulation and Interface Conditions

2.1 SPH framework

In the Lagrangian description, flow controlled partial differential equations of Navier-Stokes for fluids can be expressed as

$$\frac{\mathrm{d}\rho_i}{\mathrm{d}t} = -\rho_i \nabla \cdot \boldsymbol{v}_i \tag{1}$$

$$\rho_i \frac{\mathrm{D}\boldsymbol{v}_i}{\mathrm{D}t} = -\nabla p_i + \rho_i g + \mu \nabla^2 \boldsymbol{v}_i \tag{2}$$

where $\boldsymbol{v}_i$ is velocity, ρ_i is density, p_i is pressure, μ is viscosity coefficient and g represents the external force field. Eq. (1) is mass conservation equation and Eq. (2) is momentum conservation equation.

The theory of SPH is to utilize the form of discrete particles to characterize the successive fields and use integration to approximate the fields. For particle i at location $\boldsymbol{x}_i$,

$$\langle A(\boldsymbol{x}_i) \rangle = \sum_j m_j \frac{A_j}{\rho_j} W(\boldsymbol{x}_i - \boldsymbol{x}_j, h) \tag{3}$$

where m_j and ρ_j represent particle mass and density respectively, $W(\boldsymbol{x}_i - \boldsymbol{x}_j, h)$ is the smoothing kernel and h is the smoothing radius.

Applying Eq. (3) to the density of particle i at location $\boldsymbol{x}_i$, it yields

$$\rho_i = \sum_j m_j W(\boldsymbol{x}_i - \boldsymbol{x}_j, h) \tag{4}$$

Therefore forces between particles including pressure $\boldsymbol{f}_i^P$ and viscous force $\boldsymbol{f}_i^v$ can be represented as

$$\boldsymbol{f}_i^P = -\sum_j m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right) \nabla W_{ij} \tag{5}$$

$$\boldsymbol{f}_i^v = \mu \sum_j m_j \frac{\boldsymbol{v}_{ji}}{\rho_j} \nabla^2 W_{ij} \tag{6}$$

We employ Tait equation[6] to calculate the pressure, that is $p_i = \frac{\rho_0 c_s^2}{\gamma} \left(\left(\frac{\rho_i}{\rho_0} \right)^{\gamma} - 1 \right)$, where $\rho_0 = 1\,000$ is the rest density of fluid, $\gamma = 7$ is stiffness parameters and c_s is velocity of sound.

2.2 Fluid interface conditions

Fluid interface conditions refer to the constraint conditions of the fluid on the interface of fluid and other material. In contrast, the continuity equation (1) (incompressible fluid) and N-S equation (2) are the constraint equations inside the fluid. Fluid interfaces can be roughly divided into fluid and air (or vacuum) interface which is also called free surface, and the interface of fluid and solid, which is also called fluid-solid interface, fluid and fluid interface. Fig. 1 shows the free surface, the fluid on the interface need to be met as

$$[p] = \gamma \kappa \tag{7}$$

where $[p] = p_f - p_a$ is the difference of the pressures of fluid and air on the free surface, γ is the tension coefficient (liquid water is 0.073), κ is the mean curvature:

$$\kappa = \frac{1}{R_1} + \frac{1}{R_2} \tag{8}$$

Among them, R_1 and R_2 are two main curvature radius at the free surface, given for the implicit surface $\phi(\boldsymbol{x}) = 0$, $\frac{1}{R_1} + \frac{1}{R_2}$ is the mean curvature. We can have the following equation

$$\frac{1}{R_1} + \frac{1}{R_2} = \nabla \cdot \left(\frac{\nabla \phi}{\| \nabla \phi \|} \right) \tag{9}$$

where $\| \nabla \phi \|$ is norm.

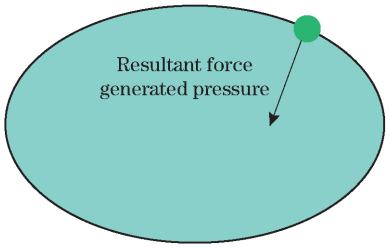


Fig. 1 Free surface(the blue one is liquid, the surrounding is air)

Fig. 2 shows the fluid-solid interface, the fluid on the interface needs to be met as

$$\boldsymbol{v}_r \boldsymbol{n} > 0 \tag{10}$$

where $\boldsymbol{v}_r = \boldsymbol{v}_f - \boldsymbol{v}_s$ is the relative velocity between fluid and solid at the interface, and $\boldsymbol{n}$ is a normal vector of the interface. The physical meaning of the formula is that the fluid in the flow should not penetrate the fluid-solid interface.

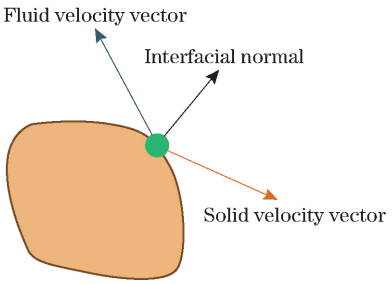


Fig. 2 Fluid and solid interface(Tan represents solid, blue represents fluid)

3 Surface Feature Model

3.1 Surface tension

In the particle-based fluid simulation, the surface tension model usually adopts the method based on color field, any particle i is assigned a color field value c_i, the same fluid particles have the same color of the field value. Take c_i into the SPH interpolation formula and we can get the color field interpolation formula as

$$c_i = \sum_j \frac{m_j c_j}{\rho_j} W_{ij} \tag{11}$$

The normal vector of surface calculated by $\boldsymbol{n} = \nabla c$, can make the normal vector of the surface points to the interior of the fluid. And the surface curvature can be measured by the divergence of the normal vector. Its form is expressed as

$$\kappa = \frac{-\nabla^2 c}{|\boldsymbol{n}|} \tag{12}$$

The surface tension of fluid can be structured by the normal vector and the surface curvature as

$$F^s = \sigma\kappa\boldsymbol{n} = -\sigma\nabla^2 c \frac{\boldsymbol{n}}{|\boldsymbol{n}|} \tag{13}$$

Although the model achieves the effect of surface tension through the color field easily, it has some shortcomings. First, less neighbor particles can cause a calculation error of $-\nabla^2 c$. Besides, the calculation of the second derivative is sensitive to the particle's disorder.

Eq. (11) and Eq. (12) only consider the cohesion, can neither show the effect of surface tension truly nor guarantee the minimum of surface area. Therefore, this section uses the surface tension model, which is similar to the method proposed by Akinci et al.[8], considering molecular poly-repulsion as well as the minimum of surface area.

First, the cohesion force model considers the effect of attraction and repulsion, similar to the intermolecular forces, making the particles attract each other when they are far away, or repelling each other when they are too close. This avoids force becoming too rigid, and it avoids the unstable problem as well. Its form is expressed as

$$F_i^c = -\alpha m_i \sum_j m_j (\boldsymbol{x}_i - \boldsymbol{x}_j) d(|\boldsymbol{x}_i - \boldsymbol{x}_j|) \tag{14}$$

where j is the neighbor particle of i, m is the particle's quality, $\boldsymbol{x}$ is the particle's position, α is the surface tension coefficient, d is the spline function.

It is obvious that the spline function d determines the nature of the surface tension F_i^c, its action is similar to the kernel function. Becker et al.[6] use a SPH nuclear function to construct the surface tension, whose effect is not similar to the intermolecular force. So if we use Eq. (9) to calculate the surface tension, we need to construct a spline function d to show molecular inter-atomic forces.

For the smooth kernel function, we can construct the smooth function by modifying the process of continuity of the particles. Assuming that we only relate the smooth function to the polynomial of the relative distance between particles, the smooth function can be formed as a polynomial, if the influence width h is in the support of the domain

$$W(x-x',h) = W(R) = a_0 + a_1 R + a_2 R^2 + \cdots + a_n R^n \tag{15}$$

In some cases, it would be good to choose piecewise smooth functions. Because the shape of the piecewise smooth functions can be well adjusted by changing the number of segments and the location of the connection point. If the smooth function is divided into two segments, the general form is shown as

$$W(R) = \begin{cases} W_1(R), & 0 \leqslant R < R_1 \\ W_2(R), & R_1 \leqslant R < R_2 \\ 0, & R_2 < R \end{cases} \tag{16}$$

Equation itself, its first derivative and second

derivative at the connection point must be continuous. And considering the connection point conditions and the compact support condition, the form of the smooth function may be

$$W(R)=\begin{cases} b_1\ (R_1-R)^n+b_2\ (R_2-R)^n, & 0\leqslant R<R_1 \\ b_2\ (R_2-R)^n, & R_1\leqslant R<R_2 \\ 0, & R_2<R \end{cases} \tag{17}$$

We can adopt a similar way to structure the spline function d of Eq. (9). The surface tension produces repulsion, if the distance of particle is less than a certain threshold, or produces attraction within the scope of the neighbor particles, if the distance of particle is greater than a certain threshold. For comparison, this paper chooses the spline function from the method proposed by Akinci et al.[8], the form is

$$d(r)=\frac{32}{\pi h^9}\begin{cases} (h-r)^3 r^3, & \frac{h}{2}<r\leqslant h \\ 2\ (h-r)^3 r^3-\frac{h^6}{64}, & 0<r\leqslant\frac{h}{2} \\ 0, & \text{other} \end{cases} \tag{18}$$

Eq. (14) can make the surface tension force calculation Eq. (18) generate attraction and repulsion according to the size of the distance between particles. At a distance of $h/2$, the attraction achieves its maximum. And it will smoothly tend to zero as the distance gradually increases to h.

3.2 Improved surface tension

To better simulate the microscopic surface feature and show the area minimizing effect, we need to add the correction term to the above tension model. In order to avoid computing the surface curvature in an explicit way, the normal vector is expressed as follows, which is similar to the normal vector calculation method according to the color field:

$$\boldsymbol{n}_i=\alpha\sum_j \frac{m_j}{\rho_j}\nabla W(|\boldsymbol{x}_i-\boldsymbol{x}_j|) \tag{19}$$

where α is the zoom factor.

The surface tension tends to reduce the curvature, and make different discrete sampling points have a consistent direction. Besides, the curvature is proportional to the normal difference, so the modifications are expressed as

$$\boldsymbol{F}_i^f=-\beta m_i\sum_j(\boldsymbol{n}_i-\boldsymbol{n}_j) \tag{20}$$

where β is the correction coefficient. The correction term increases with the curvature. Its value is 0 at the flat areas and the interior. It avoids the standardization of the normal $\boldsymbol{n}_i$ and the explicit calculation of curvature.

Above all, the revised surface tension can be expressed as

$$\boldsymbol{F}_i^{cf}=\gamma_{ij}(\boldsymbol{F}_i^c+\boldsymbol{F}_i^f) \tag{21}$$

where γ_{ij} is the surface tension control coefficient, this section makes it $\gamma_{ij}=\frac{2\rho_0^2}{\rho_i^2+\rho_j^2}$. $\gamma_{ij}>1$ near the surface of the fluid, and $\gamma_{ij}\approx 1$ inside the fluid.

3.3 Adhesion between fluid and solid

As previously mentioned, adsorption is generated by molecular interactions of different material. In this section, adsorption is mainly aimed at the effect between fluid and solid. In this paper, we process the fluid-solid coupling simulation in the following steps. Firstly, we sample the rigid body surface as boundary particles. Then, we use the boundary treatment proposed by Akinci et al.[15] to calculate the fluid density, where the boundary particles are considered. At last, the force formula can be derived. Hence, in this section we use a adsorption force model that can be applied to the fluid-solid coupling directly[8].

$$\boldsymbol{F}_i^a=-\eta m_i\sum_k \boldsymbol{\psi}_{b_k}(\boldsymbol{x}_i-\boldsymbol{x}_k)\,y(|\boldsymbol{x}_i-\boldsymbol{x}_k|) \tag{22}$$

where k is the boundary particle, $\boldsymbol{x}$ is the position of particle, η is the adsorption parameter, $\boldsymbol{\psi}_{b_k}$ is the volume of boundary particle. y is the spline function, similar to Eq. (22), an approach to construct y is

$$y(r)=\frac{0.01}{h^5}\begin{cases} -\left(r-\frac{3}{4}h\right)^2+\frac{h^2}{16}, & \frac{h}{2}<r\leqslant h \\ 0, & \text{other} \end{cases} \tag{23}$$

Using the boundary treatment methods can

solve the adhesion and clustering effect in border of fluid particle effectively, the above type just imposed the adsorption effect on the particles from $h/2$ to h to attract each other. You can see that the adsorption is symmetrical, namely $\boldsymbol{F}_k^a = -\boldsymbol{F}_i^a$.

4 Surface Feature Simulation Based on IISPH

Our surface feature model can minimize the surface area, prevent clustering, and realize the surface tension and adsorption effect. However, when the tension or adhesion is the main force, the time step will be limited. PCISPH and adaptive time step[15] have been used to make the problem easy to solve, but there are still numerical instability and low efficiency issues. To improve the numerical stability and computational efficiency of the surface feature simulation, this section presents the surface feature simulation algorithm based on IISPH.

The framework of SPH fluid simulation has two categories. One is based on the equation of state, such as WCSPH, PCISPH[6,16] etc. Another is based on the pressure projection, such as ISPH[17-20]. The thought of ISPH contains the following steps. First, we use the force except pressure to predicts the intermediate velocity of particle. Then, we solve the pressure Poisson equation. Finally, we use the pressure to obtain other physical quantities. However this kind of methods are usually time-consuming. Therefore, Ihmsen et al.[21] proposed IISPH method that combined the continuity equation discretized by SPH and the symmetric SPH pressure to get the pressure Poisson equation (PPE). Then use relaxation Jacobi iteration to get the pressure. IISPH can use a large time step and make the density deviation less than 0.01% so that it improves the efficiency and stability of the simulation significantly.

IISPH method discretizes the fluid continuity equation $\frac{D\rho}{Dt} = -\rho\nabla \cdot v$ into

$$\frac{\rho_i(t+\Delta t)-\rho_i(t)}{\Delta t} = \sum_j m_j[\boldsymbol{v}_i(t+\Delta t)-\boldsymbol{v}_j(t+\Delta t)]\nabla W_{ij}(t) \tag{24}$$

The speed difference $\boldsymbol{v}_i(t+\Delta t)-\boldsymbol{v}_j(t+\Delta t)$ in Eq. (24) depends on the pressure force $\boldsymbol{F}^p(t)$ at time t, the pressure force is dependent on the pressure $p(t)$.

Using the semi implicit Euler integral updating speed, we can express the speed in Eq. (24) as

$$\boldsymbol{v}_i(t+\Delta t) = \boldsymbol{v}_i(t) + \Delta t \frac{\boldsymbol{F}_i^o(t)+\boldsymbol{F}_i^p(t)}{m_i} \tag{25}$$

where $\boldsymbol{F}_i^o(t)$ means the forces in addition to the pressure force, including surface tension, adsorption force, viscous force, gravity, etc.

According to $\boldsymbol{F}_i^o(t)$, we forecast the intermediate speeds of fluid particles are

$$\boldsymbol{v}_i^*(t+\Delta t) = \boldsymbol{v}_i(t) + \Delta t \frac{\boldsymbol{F}_i^o(t)}{m_i} \tag{26}$$

According to the form of Eq. (24), the intermediate speeds can deduce intermediate densities of fluid particles, which are as follows

$$\rho_i^*(t+\Delta t) = \rho_i(t) + \Delta t \sum_j m_j[\boldsymbol{v}_i^*(t+\Delta t)-\boldsymbol{v}_j^*(t+\Delta t)]\nabla W_{ij}(t) \tag{27}$$

Make $\rho_i(t+\Delta t) = \rho_0$, use the intermediate density $\rho_i^*(t+\Delta t)$ instead of $\rho_i(t)$. Then take them into Eq. (24), so Eq. (24) can be expressed as

$$\frac{\rho_0-\rho_i^*(t+\Delta t)}{\Delta t} = \sum_j m_j[\boldsymbol{v}_i^*(t+\Delta t)-\boldsymbol{v}_j^*(t+\Delta t)]\nabla W_{ij}(t) \tag{28}$$

After decreasing, the difference of the density is

$$\Delta t^2 \sum_j m_j\left(\frac{\boldsymbol{F}_i^p(t)}{m_i}-\frac{\boldsymbol{F}_j^p(t)}{m_j}\right)\nabla W_{ij}(t) = \rho_0 - \rho_i^*(t+\Delta t) \tag{29}$$

Next, we need to solve the above formula and get the pressure value in a certain range of density fluctuations. Taking the pressure term into the above formula can get the following system of linear equations

$$\sum_j a_{ij}p_j = \rho_0 - \rho_i^*(t+\Delta t) \tag{30}$$

It can be seen that only pressure values are unknown in this system.

Eq. (30) can be solved by the relaxation Jacobi iteration, namely

$$p_i^{l+1}=(1-\omega)p_i^l+\omega\frac{1}{a_{ii}}\Big[\rho_0-\rho_i^*(t+\Delta t)-\sum_{j\neq i}a_{ij}p_j^l\Big] \tag{31}$$

where l is the number of iterations, ω is the coefficient of relaxation.

In order to calculate Eq. (31), a_{ii} and $\sum_{j\neq i}a_{ij}p_j^l$ need to be determined. So the displacement of pressure is expressed as

$$\Delta t^2\frac{\boldsymbol{F}_i^p(t)}{m_i}=-\Delta t^2\sum_j m_j\Big(\frac{p_i}{\rho_i^2}+\frac{p_j}{\rho_j^2}\Big)\nabla W_{ij}=\Big(-\Delta t^2\sum_j\frac{m_j}{\rho_i^2}\nabla W_{ij}\Big)p_i+\sum_j\Big(-\Delta t^2\frac{m_j}{\rho_j^2}\nabla W_{ij}\Big)p_j=\boldsymbol{d}_{ii}p_i+\sum_j\boldsymbol{d}_{ij}p_j \tag{32}$$

where $\boldsymbol{d}_{ii}=-\Delta t^2\sum_j\frac{m_j}{\rho_i^2}\nabla W_{ij}$, $\boldsymbol{d}_{ij}=-\Delta t^2\frac{m_j}{\rho_j^2}\nabla W_{ij}$.

Substitute Eq. (32) into Eq. (29) and k identifies neighbor of j

$$\rho_0-\rho_i^*(t+\Delta t)=p_i\sum_j m_j(\boldsymbol{d}_{ii}-\boldsymbol{d}_{ij})\nabla W_{ij}+\sum_j m_j\Big(\sum_j\boldsymbol{d}_{ij}p_j-\boldsymbol{d}_{jj}p_j-\sum_{k\neq i}\boldsymbol{d}_{jk}p_k\Big)\nabla W_{ij} \tag{33}$$

a_{ii} and p_i^{l+1} can be expressed as

$$a_{ii}=\sum_j m_j(\boldsymbol{d}_{ii}-\boldsymbol{d}_{ij})\nabla W_{ij} \tag{34}$$

$$p_i^{l+1}=(1-\omega)p_i^l+\omega\frac{1}{a_{ii}}(\rho_0-\rho_i^*(t+\Delta t)-\sum_j m_j\Big(\sum_j\boldsymbol{d}_{ij}p_j^l-\boldsymbol{d}_{jj}p_j^l-\sum_{k\neq i}\boldsymbol{d}_{jk}p_k^l\Big)\nabla W_{ij}\Big) \tag{35}$$

Combining the above IISPH with the previous surface feature model, an efficient and stable algorithm is shown as follows.

Algorithm 1 Surface feature simulation based on IISPH

```
while animating do
  for each particle i do
    search neighbor particles of i, get N_i(t)
  for each particle i do
    calculate the density ρ_i(t)
    calculate the resultant force in addition to the pressure force F_i^p(t)
    calculate intermediate speed v_i^*
    calculate d_ii
for each particle i do
    calculate intermediate density ρ_i^*(t)
  set the initial value p_i^0=0.5p_i(t-Δt)
  calculate a_ii
l=0
while ρ_avg^err>η or l<2 do
for each particle i do
  calculate Σ_j d_ij p_j^l
for each particle i do
  calculate ρ_i^{l+1}
  calculate p_i^{l+1}
  p_i(t)=p_i^{l+1}
  calculate ρ_i^err=ρ_i^{l+1}-ρ_0
  calculate ρ_avg^err=(1/n) Σ ρ_i^err
  l=l+1
for each particle i do
v_i(t+Δt)=v_i^*+Δt F_i^p(t)/m_i
x_i(t+Δt)=x_i(t)+Δt v_i(t+Δt)
```

5 Implement and Results

This section verifies the effectiveness of our method. The platform is the Intel Xeon E5-2687W v4 (eight nuclear, 3.0 GHz, 20 MB Cache) with 72 GB memory. The surface construction algorithm and simulation algorithm are implemented in C++ language using multithreading technology that uses space background grid hash lookup to search neighbor particles. We employed the OpenGL 3D graphics library to achieve the real-time display simulation, and used Blender to implement offline rendering. All experimental scenario's parameters in this section are shown in Tab. 1.

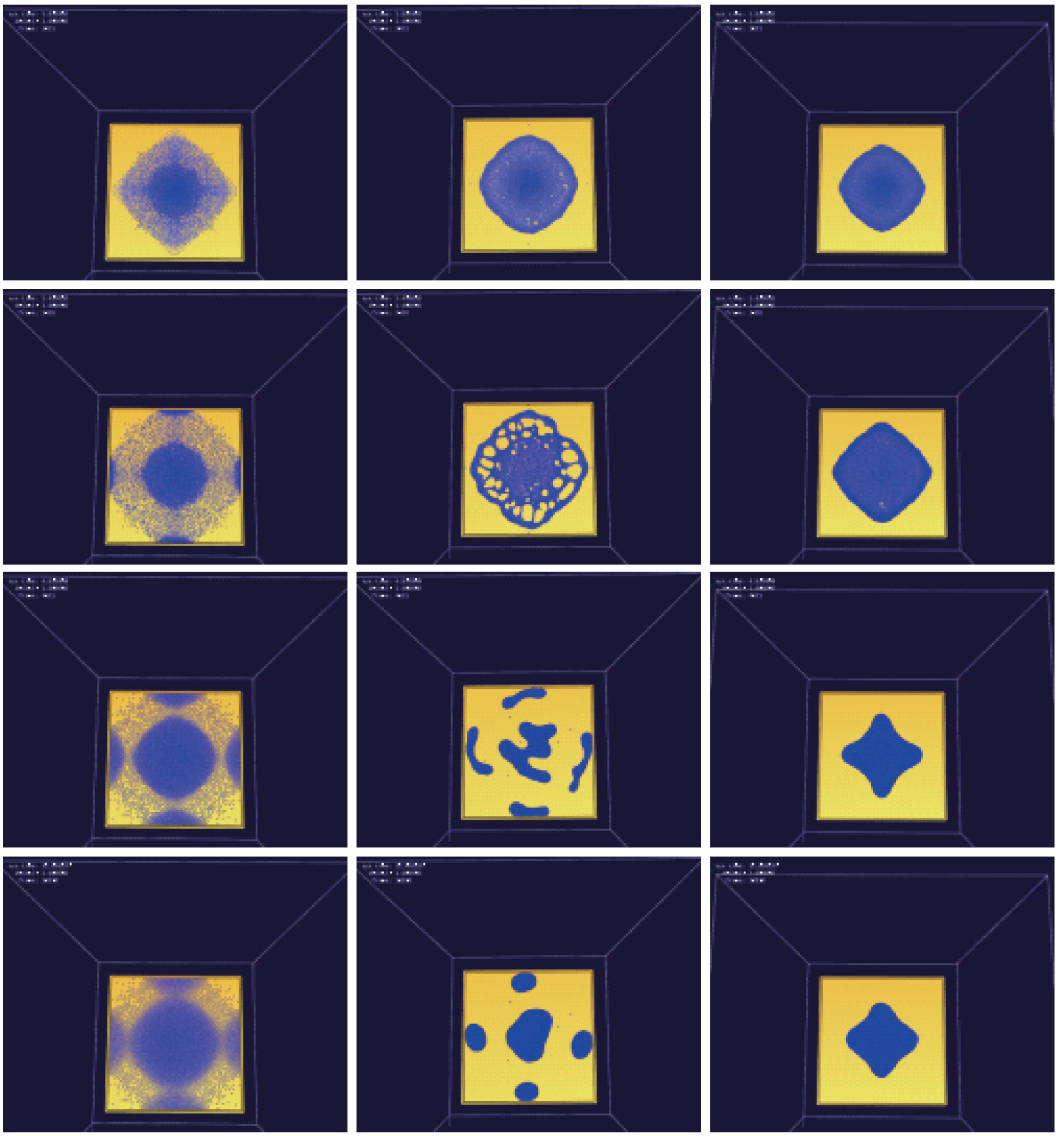
Tab. 1 Setting and statistics of simulation

Item	Value
Simulation domain size	8 m×8 m×8 m
Fluid density	1 000 kg/m^3
Smooth and kernel function	Cubic spline function
Smooth radius	0.2 m
Width of the fluid particles	0.1 m

Fig. 3 shows the experiment of a square water flow on the tablet. At the beginning, the falling water contacts with flat gradually, then the

fluid tiles along the flat gradually. Because of surface tension and the function of adsorption force, fluid tends to become static and forming single-layer fluid on tablet. Fig. 3a is IISPH method without surface tension and adsorption. It can be seen that if we ignore the function of surface tension and adsorption force, particles are in a relatively decentralized state, and the microscopic characteristics of the fluid are worse than the latter two columns.

Fig. 3b shows the results of Akinci et al. [8], Fig. 3c shows our result. It can be found that under the same condition, if particles collide violently, our method is more stable than the method of Akinci et al. [8], and constringe much faster (It can be seen from the last two lines in Fig. 3). This shows that when dealing with the bigger and more violent scenes, our simulating model will be more stable and faster. In addition, the shape of the fluid under a steady state is mainly determined by the surface tension and the adsorption force, which makes the curvature of different direction become consistent, and minimizes the surface area. It can be seen in Fig. 3 that our method is better than the method proposed by Akinci et al. The effect of minimizing the surface is better, and the whole simulation process is more stable.



(a) No surface tension and adsorption (b) Akinci's method (c) Our method

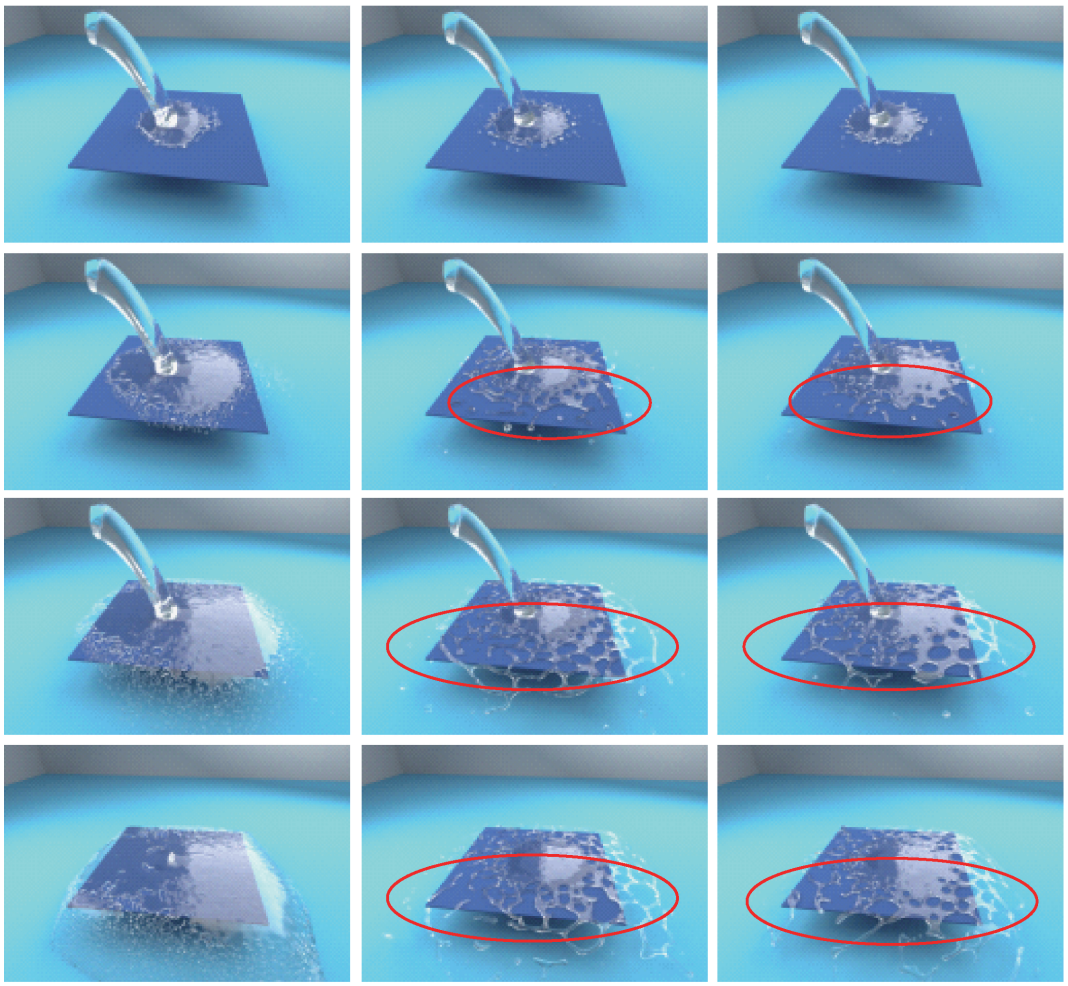
Fig. 3 Square water flow on the tablet

Tab. 2 shows the comparison of spilled water on board. It can be seen that our method can shorten the operation time of simulation and improve efficiency obviously under the same scene parameters.

Tab. 2 Statistics of spilled water on board

Method	Number of particles	Comp time/s
Akinci's method	32 751	794
Our method	32 571	442

Fig. 4 shows the experiment of the water impacting the plate. This method can realize the effect of water flow by adding the single layer fluid at different time points, and does not show the experimental time. Similar to the previous two experiments, the experiment compares Fig. 4a with Figs. 4b and 4c to show that the microscopic characteristics of the fluid surface are more realistic when the surface tension and adsorption force of the fluid are added. Compared with method proposed by Akinci et al., the fluid stability is better and the fluid splash effect is not intense (the last two figures of the second row and the third line). In addition, it can also be further verified by the last two graphs of the fourth row that the fluid region minimization effect is better and has a greater fluid adhesion area.



(a) No surface tension and adsorption (b) Akinci's method (c) Our method

Fig. 4 Spilled water on board

6 Conclusion

A surface feature model based on IISPH is proposed for fluid simulation. This method improves Akinci's surface tension model by combining it with IISPH. It simulates fluid surface ten-

sion and attraction realistically, and improves the numerical stability and computational efficiency. The experimental results demonstrate that the method can simulate the surface tension and the adsorption well in a variety of scenarios. Compared with previous methods, our method has high efficiency and stability, and realizes better micro characteristics, such as minimizing the fluid surface. In addition, the surface tension and adsorption can keep a good performance in larger and more intense scenes.

References:

[1] Brackbill J U, Kothe D B, Zemach C. A continuum method for modeling surface tension[J]. Journal of Computational Physics, 1992, 100(2): 335-354.

[2] Morris J P. Simulating surface tension with smoothed particle hydrodynamics[J]. International Journal for Numerical Methods in Fluids, 2000, 33(3):333-353.

[3] Müller M, Charypar D, Gross M. Particle-based fluid simulation for interactive applications[C]// Proceedings of the 2003 ACM SIGGRAPH/ Eurographics Symposium on Computer Animation. Aire-la-Ville, Switzerland: Eurographics Association, 2003: 154-159.

[4] Müller M, Solenthaler B, Keiser R, et al. Particle-based fluid-fluid interaction[C]// Proceedings of the 2005 ACM SIGGRAPH/Eurographics Symposium on Computer Animation. Aire-la-Ville, Switzerland: Eurographics Association, 2005:591-594.

[5] Nugent S, Posch H A. Liquid drops and surface tension with smoothed particle applied mechanics [J]. Physical Review E, 2000, 62(4): 4968-4975.

[6] Becker M, Teschner M. Weakly compressible SPH for free surface flows[C]// Proceedings of the 2007 ACM SIGGRAPH/Eurographics Symposium on Computer Animation. Aire-la-Ville, Switzerland: Eurographics Association, 2007: 209-217.

[7] Tartakovsky A, Meakin P. Modeling of surface tension and contact angles with smoothed particle hydrodynamics[J]. Physical Review E, 2005, 72(2): 254-271.

[8] Akinci N, Akinci G, Teschner M. Versatile surface tension and adhesion for SPH fluids[J]. ACM Transactions on Graphics, 2013, 32(6): 182. http:// dx. doi. org/10. 1145/2508363. 2508395.

[9] Clavet S, Beaudoin P, Poulin P. Particle based viscoelastic fluid simulation[C]// Proceedings of the 2005 ACM SIGGRAPH/Eurographics Symposium on Computer Animation. New York, USA: ACM, 2005:219-228.

[10] Yu J, Wojtan C, Turk G, et al. Explicit mesh surfaces for particle based fluids[J]. Computer Graphics Forum, 2012, 31(2):815-824.

[11] Steele K, Cline D, Egbert P K, et al. Modeling and rendering viscous liquids[J]. Computer Animation & Virtual Worlds, 2004, 15(3-4): 183-192.

[12] Schechter H, Bridson R. Ghost SPH for animating water[J]. ACM Transactions on Graphics, 2012, 31(4): 61. http:// doi. acm. org/10. 1145/2185520. 2185557.

[13] He X, Liu N, Wang G, et al. Staggered meshless solid-fluid coupling [J]. ACM Transactions on Graphics, 2012, 31(6): 439-445.

[14] Liu G R, Liu M B. Smoothed particle hydrodynamics: a meshfree particle method[M]. Singapore: World Scientific, 2004.

[15] Akinci N, Ihmsen M, Akinci G, et al. Versatile rigid-fluid coupling for incompressible SPH[J]. ACM Transactions on Graphics, 2012, 31(4): 62. http:// doi. acm. org/10. 1145/2185520. 2185558.

[16] Solenthaler B, Pajarola R. Predictive-corrective incompressible SPH[J]. ACM Transactions on Graphics, 2009, 28(3): 341-352.

[17] Bodin K, Lacoursiere C, Servin M. Constraint fluids [J]. IEEE Transactions on Visualization & Computer Graphics, 2012,18(3): 516-526.

[18] Cummins S J, Rudman M. An SPH projection method[J]. Journal of Computational Physics, 1999,152(2): 584-607.

[19] Premžoe S, Tasdizen T, Bigler J, et al. Particle-based simulation of fluids [J]. Computer Graphics Forum, 22(3): 401-410.

[20] Losasso F, Talton J O, Kwatra N, et al. Two-way coupled SPH and particle level set fluid simulation [J]. IEEE Transactions on Visualization & Computer Graphics, 2008, 14(4): 797-804.

[21] Ihmsen M, Cornelis J, Solenthaler B, et al. Implicit incompressible SPH[J]. IEEE Transactions on Visualization & Computer Graphics, 2014, 20(3): 426-435.

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