Convergent Turbulence Refinement toward Irrotational Vortex

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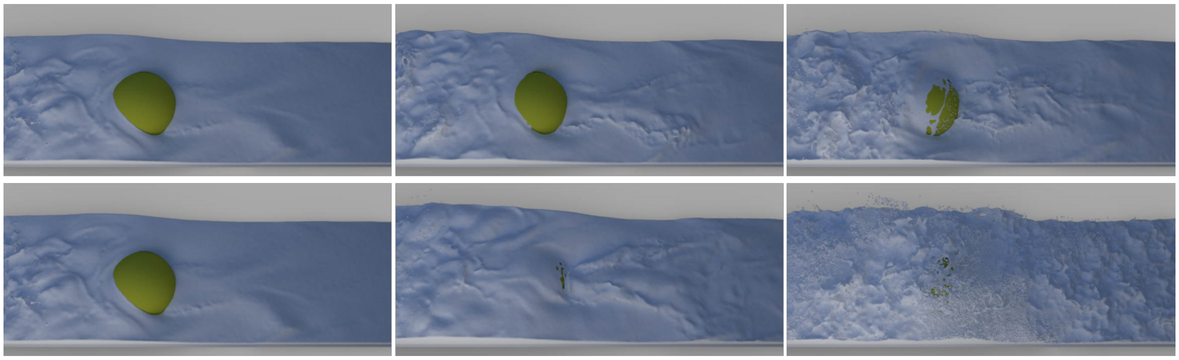

Figure 1: Breaking dam with 94K fluid particles(The radius of them is 0.1m) injected per second using MP method[Bender et al. 2018] (second row) and our method (first row) simulated with different coefficients (left to right): $\alpha = 0, 0.2, 0.6$.

ABSTRACT

We proposed a detail refinement method to enhance the visual effect of turbulence in irrotational vortex. We restore the missing angular velocity from the particles and convert them into linear velocity to recover turbulent detail due to numerical disspation.

CCS CONCEPTS

• **Computing methodologies → Physical simulation**; *Real-time simulation*; *Interactive simulation.*

KEYWORDS

turbulence simulation, SPH, vortex-based method

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1 INTRODUCTION

Fluid simulation is a hot topic in computer graphics, which was first introduced by Stam in 1999 [Stam 1999]. Recently, higher demands for fluid simulation is put forward, such as the simulation of turbulence with complex surface details. Over the last years, turbulent fluid simulation has been more and more popular.

There are two kinds of method that used to increase the resolution of turbulent fluid, that is, up-res method and vortex-based method. The main idea of up-res method is up-sampling under coarse discretization, such as CNN, example-based method [Sato et al. 2018]. However, Up-res methods act as a post processing step which can be combined with our method.

Vortex-based method aims at creating and preserving turbulence through vorticity field. It could be divided into two main categories, vorticity confinement method and Lagrangian vortex method. Vorticity confinement was introduced in computer graphics by Fedkiw [Fedkiw et al. 2001]. The core idea is recover existing vortexes and enhanced them by adding a new force. Lagrangian vortex methods build on vorticity representation of Navier-Stokes equations. They are naturally divergence-free and inherently immune to numerical dissipation. They could be implemented by surface[Weißmann and Pinkall 2010], filaments[Eberhardt et al. 2017] and particles[Bender et al. 2018]. Our method can be regarded as a vortex particle method. When higher efficiency is desired in particle-based approach, particles' size would become larger. The inertia tensor being absent from the equation could lead to severe numerical dissipation. To

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solve this problem, we proposed a turbulence refinement method to recover the linear velocity from missing angular velocity to enhance turbulent detail.

2 OUR APPROACH

For computing the curl of a field $\boldsymbol{A}_i$ in SPH, we apply the difference curl formulation $(\nabla \times \boldsymbol{A}_i)^{diff} = \frac{1}{\rho_i}\sum_j m_j(\boldsymbol{A}_i - \boldsymbol{A}_j) \times \nabla_i W_{ij}$, where ρ_i is the density at the location of particle i, m is the mass of each particle, and W is the smoothed kernel in SPH approach, we use spline kernel in our experiments. It is used to derive angular velocity $\boldsymbol{\omega}$. An extra relax factor α is added, which enable users to decide how rough the turbulence they desire. So the angular velocity for particle i at k_{th} time step is:

$$\boldsymbol{\omega}_i^{(k)} = \boldsymbol{\omega}_i^{(k-1)} - \alpha(\boldsymbol{\omega}_i^{(k-1)} - \frac{(\nabla \times \boldsymbol{v}_i^{(k)})^{diff}}{2}) \tag{1}$$

where α can be set between 0 to 1. According to this equation, the angular velocity that be used to refine linear velocity field is:

$$\delta\boldsymbol{\omega}_i^{(k)} = \boldsymbol{\omega}_i^{(k)} - \boldsymbol{\omega}_i^{(k-1)} = \alpha(\boldsymbol{\omega}_i^{(k-1)} - \frac{(\nabla \times \boldsymbol{v}_i^{(k)})^{diff}}{2}) \tag{2}$$

Using Eqn 2 we can successfully recover the rotational kinetic energy and convert it to angular velocity.

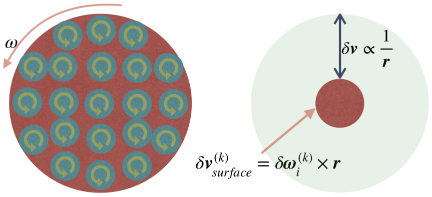

Figure 2: Velocity for each point at the particle surface

To refine linear velocity using the difference of angular velocity, we need to convert $\delta\boldsymbol{\omega}_i^{(k)}$ into $\delta\boldsymbol{v}_{i\to j}^{(k)}$ using irrotational refinement model. In our model, we take the space inside particle radius as rigid body rotation, and other space inside support radius as irrotational flow. Since we can get $\delta\boldsymbol{\omega}_i^{(k)}$ for each particle, we treat each particle as a rigid sphere, in this case we can obtain velocity for each point at the particle surface:

$$\delta\boldsymbol{v}_{surface}^{(k)} = \delta\boldsymbol{\omega}_i^{(k)} \times \boldsymbol{r} \tag{3}$$

By inversely refining the neighbor particles within the supporting radius, we can adjust the linear velocity for every particle:

$$\delta\boldsymbol{v}_{i\to j}^{(k)} = \frac{\|\boldsymbol{x}_{ij}\|}{\|\boldsymbol{r}\|}\delta\boldsymbol{v}_{surface}^{(k)} \tag{4}$$

3 RESULTS AND DISCUSSION

We show the results of our method compared with the MP method and the IISPH method [Ihmsen et al. 2013] (See attached video).

Figure 1 shows the simulation of breaking dam. Under the same coefficient, the turbulence effect behind the obstacle using our method is more obvious. In addition, the numerical instability begins to appear with MP method when the coefficient is increased, especially the coefficient value exceeds 0.6. This experiment shows that our method is more stable and does not add too much energy.

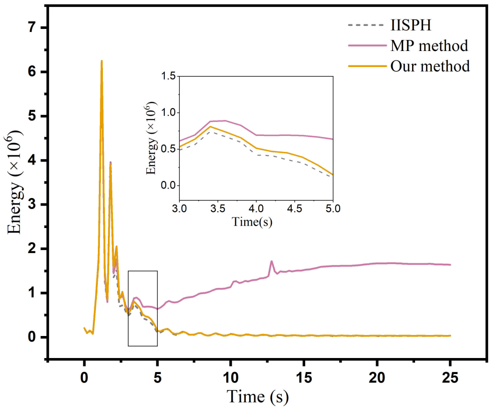

Figure 3: Energy comparison of IISPH, MP method and our method with $\alpha = 0.6$ in the breaking dam with a cylindric obstacle (4.57 M fluid particles).

Figure 3 depicts the total energy comparison of IISPH, MP method and our method. MP method adds more energy to the simulation, which results in faster flow and unstable turbulence results.

4 CONCLUSION

We present a convergent turbulence refinement method that restoring the angular velocity to recover turbulent effect. This method can significantly reduce numerical dissipation. Experiment shows this method enhances the turbulent effects compared to classical method without adding more energy than numerical disspation.

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