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Surface Tension Model Based on Implicit Incompressible Smoothed Particle Hydrodynamics for Fluid Simulation

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Abstract In order to capture stable and realistic microscopic features of fluid surface, a surface tension and adhesion method based on implicit incompressible SPH (smoothed particle hydrodynamics) is presented in this paper. It gives a steady and fast tension model and can solve the problem of not considering adhesion. Molecular cohesion and surface minimization are considered for surface tension, and adhesion is added to show the microscopic characteristics of the surface. To simulate surface tension and adhesion stably and efficiently, the surface tension and adhesion model is integrated to an implicit incompressible SPH method. The experimental results show that the method can better simulate surface features in a variety of scenarios compared with previous methods and meanwhile ensure stability and efficiency.

Keywords computer animation, fluid simulation, implicit incompressible smoothed particle hydrodynamics (IISPH), surface tension

1 Introduction

Surface tension is one of the most common and important physical characteristics to reveal microscopic effects of fluids. With the increasing demand for details and realistic effects of fluid simulation, the simulation of surface tension is becoming more and more attractive. Surface tension is produced by the cohesion among the fluid molecules. Because molecules in fluid interior are acted by forces from all directions, they reach mechanical equilibrium. While the resultant force of fluid molecules on the surface is not zero, it points to fluid interior and generates the surface tension. In addition, according to the Laplacian law, surface tension is capable of minimizing surface area. For example, the droplets shrink into a sphere when they are not subjected to external force. Using SPH (smoothed particle hydrodynamics) method to simulate surface tension realistically, however, is still a challenging problem. The computed density of particles at the fluid-air interface is lower than its real value, which is caused by lacking neighbor particles. It results in the generation of negative pressure and causes particle clustering. In addition, the simulation of microscopic characteristics takes a large amount of calculation, and has the problem of time step restriction and numerical instability. For these reasons, we propose a surface tension method based on IISPH (implicit incompressible SPH) which improves the computational efficiency and stability, and obtains a good surface tension and adsorption effect.

Nowadays, surface tension models are mainly divided into two categories: macroscopic model and microscopic model. The macroscopic model is also known as continuous surface force model (CSF)[1], which is a kind of color field method. The value of the color field changes sharply at two-phase interfaces[1-4]. The

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macroscopic model first interpolates the value of particles' color field by SPH formulas. Next, the normal vector of the surface is calculated according to the color field gradient. Then surface curvature and surface tension could be constructed. But the surface's normal vector usually has deviations due to the SPH gradient formula, especially when the surface has sharp corners and few particles. This results in a large computational error in surface curvature. Meanwhile the macroscopic model creates an asymmetric force that is applied to fluid particles, which could not ensure momentum conservation.

The microscopic model generates surface tension by the cohesion among particles which is imitating molecular attraction[5-7]. Compared with the macroscopic model, the microscopic model avoids the calculation of color field's second derivative and surface curvature which are very sensitive to particles' derangement. Moreover, the microscopic model is simple to implement and more efficient. Nugent and Posch[5] used van der Waals equation of state to calculate attraction pressure, and extended the calculation range of attraction to four hours to obtain stable droplet effect. Tartakovsky and Meakin[7] proposed a molecular cohesion model to produce surface tension in numerical simulation. It controls the attraction and repulsion between particles by cosine function. Becker and Teschner[6] employed SPH kernel instead of cosine function to make range of attraction within smoothing length h. Moreover, the cohesion models proposed by both Tartakovsky and Meakin[7] and Becker and Teschner[6], showed repulsive force in short range and attraction in long range.

However, the micro and macro models cannot well simulate large surface tension and the surface curvature minimum effects at the same time. In addition, they may cause problems such as particle clustering and momentum violation. Akinci *et al.*[8] presented a cohesion-repulsion model that can better deal with those problems. They also considered the adhesion between fluid and solid in their model. Because the stability and efficiency is still unsatisfactory, we combine the implicit incompressible SPH method (IISPH) with the improved surface tension and adsorption force on the basis of Akinci *et al.*'s[8]. Our method can better simulate fluid surface's microscopic features and meanwhile improve the numerical stability and computational efficiency.

2 Related Work

Since the surface tension has an important effect on the details of fluid simulation, many studies have explored the surface tension methods that are appropriate for fluid simulation. In SPH fluid simulation scope, early methods minimize the surface curvature using forces to achieve the surface tension effects[2-3]. However, these methods have some problems. For instance, the curvature calculation is very sensitive to particle disorder, and the external force applied to fluid particles is asymmetrical and does not meet momentum conservation. To this end, researchers proposed only using cohesion of adjacent fluid particles at the molecular level to solve the problems of early surface tension model[6-7]. However, only using cohesion between fluid particles cannot ensure the surface minimization. What is worse, it results in unreal flow patterns when the surface tension is large. Clavet *et al.*[9] proposed a method that produces surface tension by attraction. The proposed double density relaxation method can achieve strong surface tension effects. But it is not suitable for simulating low viscous liquids. Yu *et al.*[10] proposed a surface tension method based on SPH. The curvature is estimated on the surface mesh and the surface tension is applied to the adjacent fluid particles surrounded by grids. But this surface tracking method may not be able to detect isolated fluid blocks so that the surface tension effect cannot be exhibited in these areas. Besides, the surface tension effect depends on the resolution of tracking grids. Akinci *et al.*[8] skillfully created surface tension by particles' interaction. Their method can handle large surface tension while maintaining momentum conservation. It generates repulsion for particles too close and prevents particle cluster problem of free surface without additional operation.

The adsorption force makes fluids attracted by other substances. Steele *et al.*[11] proposed a Lagrangian method for viscous fluids that achieves the fluid-to-solid adsorption effects. They used distance-dependent forces to define the adsorption properties of different type substances. But this method has difficulties in simulating high viscous fluids for using linear density cores and strict anti-penetration constraints. Subsequently, Clavet *et al.*[9] modeled the viscoelastic SPH fluid's adhesion through a distance-based attraction term. Schechter and Bridson[12] realized the fluid-to-solid adhesion effect by calculating ghost velocity at each solid particle, which is computed using the velocity of solid and the tangential component velocity of the

nearest fluid particle. Then the XSPH method based on artificial viscosity is used to calculate adhesion. In the method of He *et al.*[13], the effect of adhesion is achieved by edge effect of different slip conditions with velocity constraint. The method proposed by Akinci *et al.*[8] achieves a physically reasonable fluid-solid adhesion without additional treatments (e.g., GhostSPH). And forces are symmetrically applied to the adjacent pair of fluid-fluid particles as well as fluid-boundary particles to ensure momentum conservation. Recently, Yang *et al.*[22] proposed a refined surface tension model using pairwise forces, which accurately captures surface tension without extra forces or constraints.

3 SPH Framework

In the Lagrangian description[3], flow controlled partial differential equations of Navier-Stokes for fluids can be expressed as

$$\frac{\mathrm{d}\rho_i}{\mathrm{d}t} = -\rho_i \nabla \cdot \boldsymbol{v}_i, \tag{1}$$

$$\rho_i \frac{D\boldsymbol{v}_i}{Dt} = -\nabla p_i + \rho_i g + \mu \nabla^2 \boldsymbol{v}_i, \tag{2}$$

where $\boldsymbol{v}_i$ is the velocity, ρ_i is the density, p_i is the pressure, μ is the viscosity coefficient, and g represents the external force field. (1) is the mass conservation equation and (2) is the momentum conservation equation.

The theory of SPH[3] is to utilize the form of discrete particles to characterize the successive fields and use integration to approximate the fields. For particle i at location x_i,

$$\langle A(x_i)\rangle = \sum_j m_j \frac{A_j}{\rho_j} W(x_i - x_j, h), \tag{3}$$

where m_j and ρ_j represent the particle mass and density respectively, $W(x_i - x_j, h)$ is the smoothing kernel and h is the smoothing radius.

Applying (3) to the density of a particle i at location x_i yields the summation of density

$$\rho_i = \sum_j m_j W(x_i - x_j, h).$$

Thus, forces between particles including pressure $\boldsymbol{f}_i^P$ and viscous force $\boldsymbol{f}_i^{\boldsymbol{v}}$ can be represented as

$$\boldsymbol{f}_i^P = -\sum_j m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2}\right) \nabla W_{ij},$$

$$\boldsymbol{f}_i^v = \mu \sum_j m_j \frac{\boldsymbol{v}_{ji}}{\rho_j} \nabla^2 W_{ij}.$$

We employ Tait equation[6] to calculate the pressure, that is,

$$p_i = \frac{\rho_0 c_S^2}{\gamma} \left(\left(\frac{\rho_i}{\rho_0} \right)^{\gamma} - 1 \right),$$

where $\rho_0 = 1\,000$ is the rest density of the fluid, $\gamma = 7$ is the stiffness parameter and c_S is the velocity of sound. We use the equation in [6] to compute viscous force.

4 Surface Tension and Adhesion Model

4.1 Surface Tension

In particle-based fluid simulation, the surface tension model usually adopts the method based on color field, any particle i is assigned a color field value c_i, and the same fluid particles have the same color of the field value. Taking c_i into the SPH interpolation formula, we can get the color field interpolation formula as follows:

$$c_i = \sum_j \frac{m_j c_j}{\rho_j} W_{ij}. \tag{4}$$

The normal vector of surface calculated by $\boldsymbol{n} = \nabla c$, can make the normal vector of the surface point to the interior of the fluid. And surface curvature can be measured by the divergence of the normal vector. Its form is shown as follows:

$$\kappa = \frac{-\nabla^2 c}{|\boldsymbol{n}|}. \tag{5}$$

The surface tension of fluid can be structured by the normal vector and the surface curvature is shown as follows:

$$F^s = \sigma \kappa \boldsymbol{n} = -\sigma \nabla^2 c \frac{\boldsymbol{n}}{|\boldsymbol{n}|}.$$

Although the model achieves the effect of surface tension through the color field easily, it has some shortcomings. Firstly, less neighbor particles can cause calculation error of $-\nabla^2 c$. Besides, the calculation of the second derivative is sensitive to the particle's disorder.

Tartakovsky *et al.*[7] proposed a cohesion model to control the attraction and repulsion between the particles by the cosine function. However, this method causes clustering phenomenon. Becker *et al.*[6] used SPH kernel function instead of cosine function to create attraction. It is suitable for free surface with high curvature, and can calculate the surface tension efficiently. But due to the lack of repulsive force, it will cause severe clustering phenomenon.

(4) and (5) only consider the cohesion, and can neither show the effect of surface tension truly nor guarantee the minimum of surface area. Therefore, we use a

surface tension model that is similar to Akinci *et al.*'s[8]. It considers molecular cohesion-repulsion as well as surface minimization.

First, the cohesion force model considers the effect of attraction and repulsion, similar to the intermolecular forces, making the particles attract each other when they are far away, or repel each other when they are too close. This avoids force becoming too rigid, and it avoids unstable problems as well. Its form is shown as follows:

$$\boldsymbol{F}_i^c = -\alpha m_i \sum_j m_j(\boldsymbol{x}_i - \boldsymbol{x}_j)d(|\boldsymbol{x}_i - \boldsymbol{x}_j|), \tag{6}$$

where j is the neighbor particle of i, m is the particle's quality, $\boldsymbol{x}$ is the particle's position, α is the surface tension coefficient, and d is the spline function.

It can be seen that the spline function d determines the nature of the surface tension $\boldsymbol{F}_i^c$, and its action is similar to that of the kernel function. Becker *et al.*[6] used the SPH nuclear function to construct the surface tension, whose effect is not similar to that of the intermolecular force. Therefore if we use (6) to calculate the surface tension, we need to construct spline function d to show molecular inter-atomic forces. For comparison, we choose spline function proposed by Akinci *et al.*[8]

4.2 Surface Tension Modifications

The above surface tension model can construct the effect similar to molecular inter-atomic forces do. But in order to simulate the microscopic features of fluid surface better, and to show the area minimization effect of fluid, we still need to add the correction term to surface tension. In order to avoid calculating the surface curvature in an explicit way, the normal vector is expressed as follows, which is similar to the normal vector calculation method according to the color field:

$$\boldsymbol{n}_i = \kappa \sum_j \frac{m_j}{\rho_j} \nabla W\left(|\boldsymbol{x}_i - \boldsymbol{x}_j|\right),$$

where κ is the zoom factor.

The surface tension tends to reduce the curvature, and make different discrete sampling points have consistent direction. Besides, the curvature is proportional to normal difference, and we construct modifications as follows:

$$\boldsymbol{F}_i^f = -\beta m_i \sum_j (\boldsymbol{n}_i - \boldsymbol{n}_j),$$

where β is the correction coefficient. As we can see, the correction term increases with the increase of the curvature. Its value is 0 at the flat areas and the interior of fluid. It avoids the standardization of the normal $\boldsymbol{n}_i$ and the explicit calculation of curvature.

From the above, the revised surface tension can be expressed as:

$$\boldsymbol{F}_i^{cf} = \gamma_{ij}\left(\boldsymbol{F}_i^c + \boldsymbol{F}_i^f\right), \tag{7}$$

where γ_{ij} is the surface tension control coefficient, and we adopt $\gamma_{ij} = 2\rho_0^2/(\rho_i^2 + \rho_j^2)$. $\gamma_{ij} > 1$ represents being near the fluid surface, and $\gamma_{ij} \approx 1$ indicates being inside the fluid. Therefore, we can use γ_{ij} to enlarge the surface tension of the particles whose neighbors are insufficient.

4.3 Adhesion Between Fluid and Solid

Different from surface tension, adsorption force is generated by molecular interactions of different materials. In this subsection, the adsorption force model is mainly aimed at the effect between fluid and solid. In this paper, we process the fluid-solid coupling simulation in the following steps: 1) sampling rigid body surface as boundary particles, 2) using the boundary treatment formulas proposed by Akinci *et al.*[15] to calculate fluid density, in which the boundary particles are considered, and 3) deriving each force formula. The fluid-solid coupling method can avoid the "adhesion effect" of fluid particles at the rigid boundary effectively. Thus this subsection uses an adsorption force calculation formula which can be applied to the fluid-solid coupling method directly[8].

$$\boldsymbol{F}_i^a = -\eta m_i \sum_k \boldsymbol{\psi}_{b_k}(\boldsymbol{x}_i - \boldsymbol{x}_k)y(|\boldsymbol{x}_i - \boldsymbol{x}_k|), \tag{8}$$

where k is a boundary particle, $\boldsymbol{x}$ is the position of the particle, η is an adsorption parameter, and $\boldsymbol{\psi}_{b_k}$ is the volume of the boundary particle. y is the spline function. We construct y as follows:

$$y\left(r\right) = \frac{0.01}{h^5}\begin{cases} -\left(r - \frac{3}{4}h\right)^2 + \frac{h^2}{16}, & \text{if } \frac{h}{2} < r \leqslant h, \\ 0, & \text{otherwise.} \end{cases}$$

Due to that using the boundary treatment methods can solve the adhesion and clustering effect in border of fluid particle effectively, the above type just imposes the adsorption effect on the particles from $h/2$ to h to attract each other. You can see that the adsorption is symmetrical, namely $\boldsymbol{F}_k^a = -\boldsymbol{F}_i^a$.

5 Surface Tension and Adhesion Based on IISPH

Similar to the method proposed by Akinci *et al.*[8] our surface tension and adsorption force model can minimize the surface area, prevent clustering, and realize the surface tension and adsorption effect realistically. However, if the surface tension or the adsorption force of the particles is the main force, time step will be limited. An example is water droplets. Akinci *et al.* used the predictive-corrective in compressible SPH method to simulate fluid, and used the adaptive time step methods[15] to improve the algorithm efficiency, but there are still some problems such as numerical instability, low efficiency and so on. Therefore, in order to improve the numerical stability and computational efficiency of the surface microscopic characteristics simulation, we propose the surface tension and adsorption algorithm based on IISPH in this section.

The framework based on SPH fluid simulation has two classes. One class is based on the equation of state, such as weakly compressible SPH (WCSPH), PCISPH[6,16]. The other kind is based on the pressure projection method, such as incompressible SPH (ISPH)[17-20]. The idea of ISPH method can be divided into the following steps: 1) using the force in addition to the pressure to predict the intermediate velocity of particle, 2) solving the pressure Poisson equation, and 3) using the pressure to solve other physical quantities. But this kind of method has high computational cost. Therefore, Ihmsen *et al.*[21] proposed the IISPH method. They combined the continuity equation discretized by SPH with the symmetric SPH pressure to get the pressure Poisson equation (PPE). Then they used relaxation Jacobi iteration to solve the pressure. IISPH can use a large time step and can make the density deviation less than 0.01%. Using an implicit method in the pressure iteration, IISPH improves the efficiency and stability of the simulation significantly.

The IISPH method will firstly discrete the fluid continuity equation $D\rho/Dt = -\rho\nabla\cdot\boldsymbol{v}$ into

$$\frac{\rho_i\left(t+\Delta t\right)-\rho_i(t)}{\Delta t} = \sum_j m_j(\boldsymbol{v}_i(t+\Delta t)-\boldsymbol{v}_j(t+\Delta t))\nabla W_{ij}(t). \quad (9)$$

The speed difference $\boldsymbol{v}_i\left(t+\Delta t\right)-\boldsymbol{v}_j\left(t+\Delta t\right)$ in (9) depends on the pressure force $\boldsymbol{F}^p(t)$ at time t, and the pressure force is dependent on pressure $p(t)$.

Using semi-implicit Euler integration to update speed, we can express the speed in (9) as follows:

$$\boldsymbol{v}_i\left(t+\Delta t\right) = \boldsymbol{v}_i\left(t\right)+\Delta t\frac{\boldsymbol{F}_i^o(t)+\boldsymbol{F}_i^p(t)}{m_i},$$

where $\boldsymbol{F}_i^o(t)$ means the forces in addition to the pressure force, including surface tension, adsorption force, viscous force, gravity, etc.

According to $\boldsymbol{F}_i^o(t)$, we forecast intermediate speed of fluid particles as follows

$$\boldsymbol{v}_i^*\left(t+\Delta t\right) = \boldsymbol{v}_i\left(t\right)+\Delta t\frac{\boldsymbol{F}_i^o(t)}{m_i}. \quad (10)$$

According to the form of (9), the intermediate speed can deduce intermediate density of fluid particles as follows:

$$\rho_i^*\left(t+\Delta t\right) = \rho_i(t)+\Delta t\sum_j m_j(\boldsymbol{v}_i^*\left(t+\Delta t\right)- \boldsymbol{v}_j^*\left(t+\Delta t\right))\nabla W_{ij}(t). \quad (11)$$

We make $\rho_i\left(t+\Delta t\right)=\rho_0$, and use the intermediate density $\rho_i^*\left(t+\Delta t\right)$ instead of $\rho_i(t)$. Then we take them into (9), which can be expressed as

$$\frac{\rho_0-\rho_i^*\left(t+\Delta t\right)}{\Delta t} = \sum_j m_j\left(\boldsymbol{v}_i^*\left(t+\Delta t\right)-\boldsymbol{v}_j^*\left(t+\Delta t\right)\right)\nabla W_{ij}(t).$$

After reducing, the difference of the density is:

$$\Delta t^2\sum_j m_j\left(\frac{\boldsymbol{F}_i^p\left(t\right)}{m_i}-\frac{\boldsymbol{F}_j^p(t)}{m_j}\right)\nabla W_{ij}(t) = \rho_0-\rho_i^*\left(t+\Delta t\right). \quad (12)$$

Next, we need to solve the above formula and get pressure value in a certain range of density fluctuations. Taking the pressure term into the above formula, we can get the following system of linear equations:

$$\sum_j a_{ij}p_j = \rho_0-\rho_i^*\left(t+\Delta t\right). \quad (13)$$

It can be seen that only pressure values are unknown in this system.

(13) can be solved by relaxation Jacobi iteration, namely

$$p_i^{l+1} = (1-\omega)\,p_i^l+\omega\frac{1}{a_{ii}}\left(\rho_0-\rho_i^*\left(t+\Delta t\right)-\sum_{j\neq i}a_{ij}p_j^l\right), \quad (14)$$

where l is the number of iterations and ω is the coefficient of relaxation.

In order to calculate (14), we need to determine a_{ii} and $\sum_{j\neq i} a_{ij}p_j^l$. Thus the displacement of pressure is expressed as

$$\begin{aligned}\Delta t^2\frac{\boldsymbol{F}_i^p(t)}{m_i} &= -\Delta t^2\sum_j m_j\left(\frac{p_i}{\rho_i^2}+\frac{p_j}{\rho_j^2}\right)\nabla W_{ij}\\ &= \left(-\Delta t^2\sum_j\frac{m_j}{\rho_i^2}\nabla W_{ij}\right)p_i+\\ &\quad\sum_j\left(-\Delta t^2\frac{m_j}{\rho_j^2}\nabla W_{ij}\right)p_j\\ &= \boldsymbol{d}_{ii}p_i+\sum_j\boldsymbol{d}_{ij}p_j, \end{aligned}\tag{15}$$

where

$$\begin{aligned}\boldsymbol{d}_{ii} &= -\Delta t^2\sum_j\frac{m_j}{\rho_i^2}\nabla W_{ij},\\ \boldsymbol{d}_{ij} &= -\Delta t^2\frac{m_j}{\rho_j^2}\nabla W_{ij}.\end{aligned}$$

We substitute (15) into (12), and k identifies neighbors of j

$$\begin{aligned}&\rho_0-\rho_i^*(t+\Delta t)\\ &= p_i\sum_j m_j(\boldsymbol{d}_{ii}-\boldsymbol{d}_{ij})\nabla W_{ij}+\\ &\quad\sum_j m_j\left(\sum_j\boldsymbol{d}_{ij}p_j-\boldsymbol{d}_{jj}p_j-\sum_{k\neq i}\boldsymbol{d}_{jk}p_k\right)\nabla W_{ij}.\end{aligned}$$

a_{ii} and p_i^{l+1} can be expressed as follows:

$$a_{ii} = \sum_j m_j(\boldsymbol{d}_{ii}-\boldsymbol{d}_{ij})\nabla W_{ij},\tag{16}$$

$$\begin{aligned}p_i^{l+1} &= (1-\omega)p_i^l+\omega\frac{1}{a_{ii}}\Bigg(\rho_0-\rho_i^*(t+\Delta t)-\\ &\quad\sum_j m_j\Bigg(\sum_j\boldsymbol{d}_{ij}p_j^l-\boldsymbol{d}_{jj}p_j^l-\\ &\quad\sum_{k\neq i}\boldsymbol{d}_{jk}p_k^l\Bigg)\nabla W_{ij}\Bigg).\end{aligned}\tag{17}$$

The surface tension and adsorption force algorithm based on IISPH is shown in Algorithm 1. The algorithm's calculation process in each time step is shown as follows:

1) searching neighbor particles of each fluid particle to get its neighbor particle collection $N_i(t)$;

2) calculating the density $\rho_i(t)$ of each fluid particle and the resultant forces except pressure $\boldsymbol{F}_i^o(t) = \boldsymbol{F}_i^{cf}(t)+\boldsymbol{F}_i^a(t)+\boldsymbol{F}_i^v(t)+\boldsymbol{G}$, and calculating surface tension $\boldsymbol{F}_i^{cf}(t)$ and adhesion $\boldsymbol{F}_i^a(t)$ according to (7) and (8) respectively;

3) computing $\boldsymbol{v}_i^*$, $\boldsymbol{d}_{ii}$ according to (10) and (15) respectively;

4) calculating the intermediate density $\rho_i^*(t)$ and a_{ii} of each fluid particle according to (11) and (16) respectively, and computing the initial pressure as well; using Jacobi iteration to solve the pressure Poisson equation and calculating $\sum_j \boldsymbol{d}_{ij}p_j^l$ according to (15); computing ρ_i^{l+1} using density formula, and calculating p_i^{l+1} and updating it according to (17) until the average density deviation $\rho_{\text{avg}}^{\text{err}}$ is less than the threshold η;

5) updating the speed $\boldsymbol{v}_i(t+\Delta t)$ and position $\boldsymbol{x}_i(t+\Delta t)$ of each fluid particle.

Algorithm 1. Surface Tension and Adhesion Algorithm Based on IISPH

while animating **do**
  for each particle i **do**
    Search neighbor particles of i, get $N_i(t)$
  for each particle i **do**
    Calculate the density $\rho_i(t)$
    Calculate the resultant force in addition to the pressure force $\boldsymbol{F}_i^o(t)$
    Calculate intermediate speed $\boldsymbol{v}_i^*$
    Calculate $\boldsymbol{d}_{ii}$
  for each particle i **do**
    Calculate intermediate density $\rho_i^*(t)$
    Set the initial value $p_i^0 = 0.5p_i(t-\Delta t)$
    Calculate a_{ii}
  $l = 0$
  while $\rho_{\text{avg}}^{\text{err}} > \eta$ or $l < 2$ **do**
    for each particle i **do**
      Calculate $\sum_j \boldsymbol{d}_{ij}p_j^l$
    for each particle i **do**
      Calculate ρ_i^{l+1}
      Calculate p_i^{l+1}
      $p_i(t) = p_i^{l+1}$
      Calculate $\rho_i^{\text{err}} = \rho_i^{l+1}-\rho_0$
      Calculate $\rho_{\text{avg}}^{\text{err}} = \frac{1}{n}\sum\rho_i^{\text{err}}$
      $l = l+1$
  for each particle i **do**
    $\boldsymbol{v}_i(t+\Delta t) = \boldsymbol{v}_i^* + \Delta t\frac{\boldsymbol{F}_i^p(t)}{m_i}$
    $\boldsymbol{x}_i(t+\Delta t) = \boldsymbol{x}_i(t)+\Delta t\boldsymbol{v}_i(t+\Delta t)$

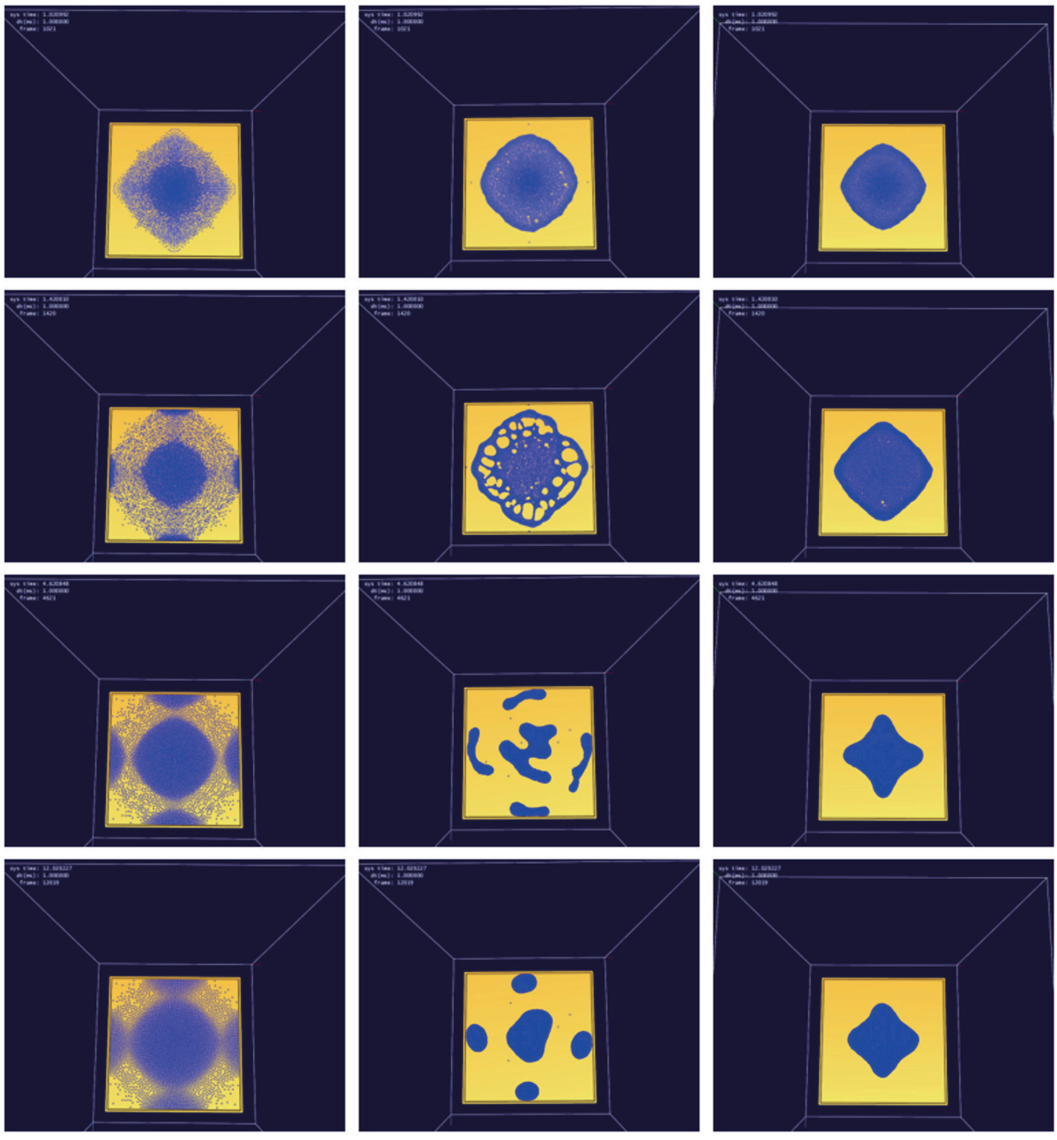
6 Implementation and Results

This section verifies the effectiveness of the proposed surface tension and adsorption force method. The program operation platform is the Intel® Xeon® E5-2687W v4 (8 core, 3.0 GHz, 20 MB cache) with 72 GB memory. The surface construction algorithm and the simulation algorithm are implemented in C++ language using multi-threading technology. The simulation algorithm uses space background grid hash lookup to search neighbor particles. We use the anisotropy method for surface reconstruction, employ OpenGL 3D graphics library to achieve real-time display simulation, and use Blender to implement offline rendering. All experimental scenarios' parameters in this section are shown in Table 1.

Table 1. Setting and Statistics of Two-Phase Breaking Dam Simulation

Item	Value
Simulation domain size	8 m × 8 m × 8 m
Fluid density	1 000 kg/m^3
Smooth and kernel function	Cubic spline function
Smooth radius	0.2 m
Width of the fluid particles	0.1 m

Fig.1 shows the experiment's particles figure of a square water flow on the tablet. At the beginning, the



(a) (b) (c)

Fig.1. Square water flow on the tablet. (a) No surface tension and adsorption. (b) Akinci *et al.*'s method[8]. (c) Our method.

falling water contacts with the tablet gradually, and then the fluid tiles along the tablet gradually. Because of the surface tension and the function of adsorption force, fluid tends to become static and form single-layer fluid on the tablet. Fig.1(a) is the results of the IISPH method without surface tension and adsorption. It can be seen that if we ignore the function of surface tension and adsorption force, particles are in a relatively decentralized state, and the microscopic characteristics of the fluid are worse than those in Figs.1(b) and 1(c).

Fig.1(b) is the results of Akinci *et al.*[8], and Fig.1(c) is our results. It can be found that under the same condition, if particles collide violently, our method is more stable than the method of Akinci *et al.*[8], and constringed much faster (it can be seen from Figs.8(b) and 8(c)). It shows that when dealing with violent scenes, our simulating model is stable and fast. In addition, the shape of the fluid under steady state is mainly decided by the surface tension and the adhesion that make the curvature of different directions become consistent, and minimize the surface area. It can be seen in the figure that our method is better than the method proposed by Akinci *et al.*[8] The effect of minimizing the surface is better, and the whole simulation process is more stable.

Fig.2 shows the rendering results of square water flow on the tablet. They correspond to the particle diagram in Fig.2. It can be seen from the figure that fluid looks more "loose" without surface tension (Fig.2(a)).

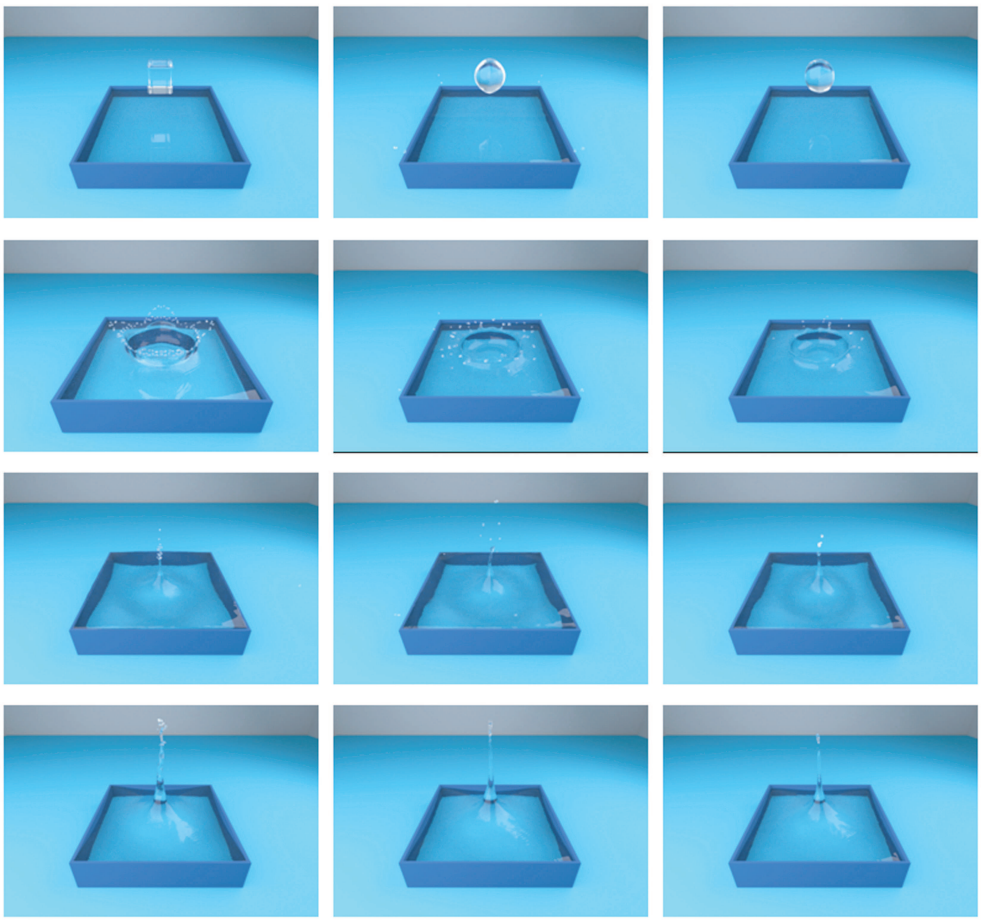
(a) (b) (c)

Fig.2. Rendering results of square water flow on the tablet. (a) No surface tension and adsorption. (b) Akinci *et al.*'s method[8]. (c) Our method.

Especially on the fluid edge, the micro effect is very poor and does not conform to the fluid characteristics in the reality. But after adding adsorption and surface tension force, detailed effect on fluid edge is improved significantly, as shown in Figs.2(b) and 2(c). Comparing Fig.2(b) with Fig.2(c), we can find that fluid surface simulated by our method is more smooth (shown by the last two pictures of the first row), and the surface minimization effect is much better (the last two pictures of the third row and the fourth row). Besides, the stability of our method is better (the last two pictures of the second row).

The experiment shown in Fig.3 is small pieces falling into the water. The "water crown" phenomenon will appear in this process. It can be seen from Fig.3(a) that the water block does not produce deformation without the surface tension. When collision occurs, the particles are in a disperse state without mutual constraints, because of the lack of surface tension. It does not conform to the reality scene. After adding the surface tension, the fluid surface effect is improved markedly, as shown in Figs.3(b) and 3(c). It can be seen from the first row of Fig.3 that the water block transforms to minimize surface area due to the effect of surface tension. Surface tension makes the water block initialized as a square tend to "sphere" in free-fall process. In addition, due to the constraint of the surface tension, fluid particles attract each other, and are controlled by attraction or repulsion to keep the balance.

Table 2 shows the comparison of experimental results including square water flow on the tablet and small pieces of wood falling into the water. It can be seen that our method can shorten the operation time of simulation and improve the efficiency obviously under the same scene parameters. (Please refer to the video jrst.wmv provided online as supplementary material at the webpage of the paper in http://www.springer.com/journal/11390.)



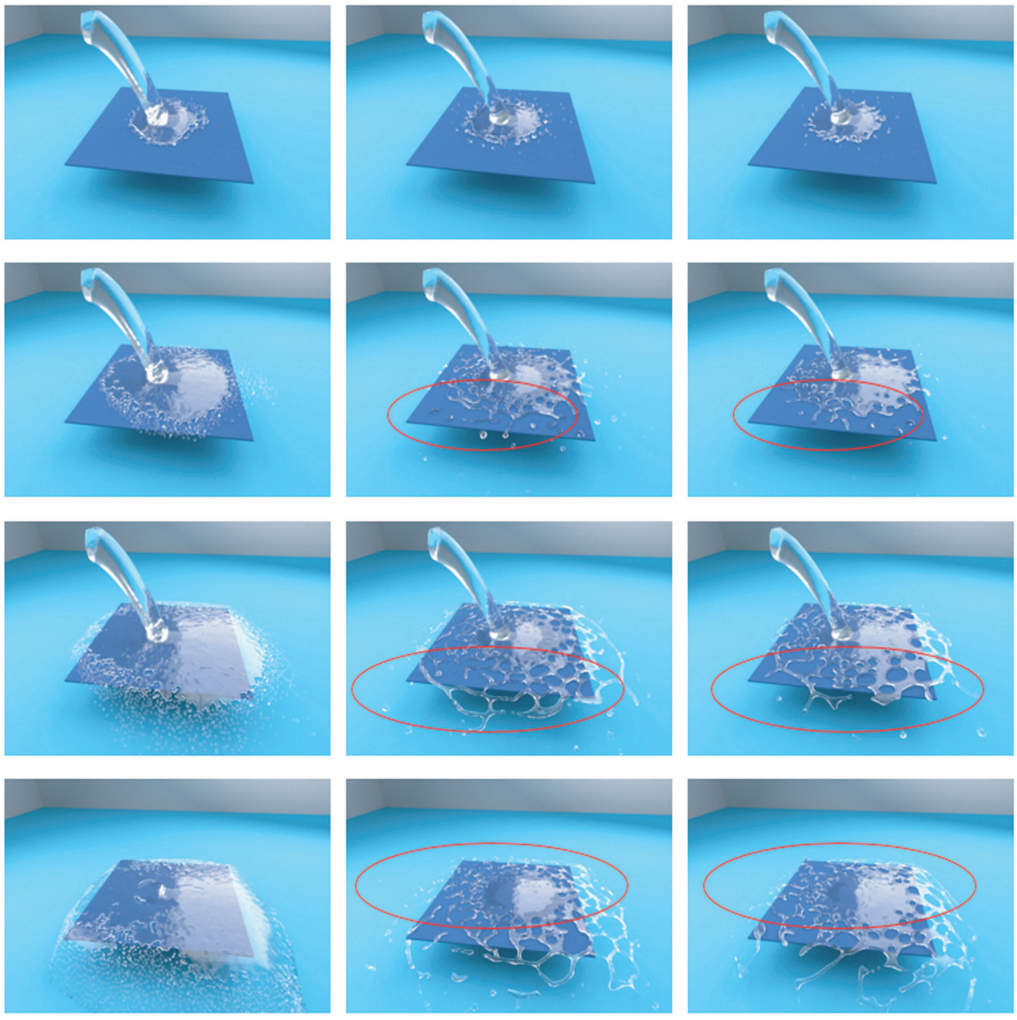
(a) (b) (c)

Fig.3. Small cube fluid falling into the water. (a) No surface tension and adsorption. (b) Akinci *et al.*'s method[8]. (c) Our method.

Table 2. Setting and Statistics of Two-Phase Breaking Dam Simulation

Scenario	Method	Number of Particles	Computing Time (s)
Square water flow on the tablet	Akinci *et al.*'s method[8]	5 491	331
	Our method	5 491	224
Small pieces falling into the water	Akinci *et al.*'s method[8]	32 751	794
	Our method	32 571	442

Fig.4 shows the experiment of the water impact plate. This method can realize the effect of water flow by adding the single layer fluid at different time points, and does not show the experimental time. Similar to the previous two experiments, the experiment compares Fig.4(a) with Fig.4(b) and Fig.4(c) to show that the microscopic characteristics of the fluid surface are more realistic when the surface tension and adsorption force of the fluid are added. Compared with Akinci *et al.*'s method[8], the fluid stability is better and the fluid splash effect is not intense (the last two pictures of the second row and the third row). In addition, it can also be further verified by the last two pictures of the fourth row that the fluid region minimization effect of our method is better and has a greater fluid adhesion area compared with Akinci *et al.*'s method[8]. (Please refer to the video jrst.wmv provided online as supplemental material at the webpage of the paper in http://www.springer.com/journal/11390.)



(a) (b) (c)

Fig.4. Spilled water on the board. (a) No surface tension and adsorption. (b) Akinci *et al.*'s method[8]. (c) Our method.

7 Conclusions

We proposed a model of surface tension and adhesion based on IISPH for fluid simulation. This method improved the surface tension method proposed by Akinci *et al.*[8] It is implemented by combining our surface tension model with the implicit incompressible SPH method (IISPH). Our method can simulate the fluid surface tension and attraction more vividly, and improve the numerical stability and computational efficiency. The experimental results showed that the method can simulate the surface tension and the adsorption of the fluid better in a variety of scenarios. Compared with Akinci *et al.*'s method[8], our method has higher efficiency and better stability, and can show micro characteristics, such as minimizing the fluid surface, in a better way. In addition, realized by combing the implicit incompressible SPH, the surface tension and adsorption can keep a good performance in a larger and more intense scene. Furthermore, it can be easily added to other SPH methods. There still has the problem in simulating multiphase fluids if the surface tension is the dominant force acting on particles. So future work will be making it suitable for multiphase fluids.

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