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Anisotropic Surface Reconstruction for Multiphase Fluids

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***Abstract*—Under particle-based framework, level set is generally defined for fluid surfaces and is integrated with marching cubes algorithm to extract fluid surfaces. In these methods, anisotropic kernels method has proven successful for reconstructing fluid surfaces with high quality. It can perfectly represent smooth surfaces, thin stream and sharp features of fluids compare to other methods. In this paper, we propose a novel approach to extend it to the simulation of multiphase fluids simulation. In order to ensure fine effects for both fluid surface and multiphase interface, we modify the calculation of original anisotropic kernels and address a binary tree strategy for reconstruction. Our method can extract fluid surfaces simply and effectively for particle-based multiphase simulation. It solved the problem of overlaps and gaps at multiphase interface that exist in traditional methods. The experimental results demonstrate that our method keep a good fluid surface and interface effects.**

Keywords—Computer animation; Fluid simulation; Surface reconstruction; Multiphase interface

I. INTRODUCTION

Fluid phenomenon exists widely in nature, daily life and industrial production. The simulation of fluid that is fluid animation has always been the significant content in the research of physically-based animation as well as computer graphics. According to the different spatial discretization way, physically-based animation can be divided into two main categories at present: mesh-based methods and mesh-free methods. In mesh-based methods, the simulation domain is discretized into mesh grids while physical attribute values on grid points (such as velocity, density) are obtained by solving the governing equations. Nevertheless, the fluid volume is discretized into sampled particles in mesh-free methods. Moreover each particle has physical properties and is advected by the governing equations. Because the mesh-free methods have the advantages of mass conservation, flexible extensibility of unbounded domains, it is easier to simulate more complex phenomena (such as spindrift, explosion, etc.). Mesh-free methods had a rapid development in recent years and have become a competitive alternative to mesh-based methods.

Interaction of fluid and fluid, also known as multiphase fluids, is very common in real life, such as water and oil mixture. At present, multiphase fluid simulation gradually attracts much attention, while the researches using particle-based methods are still rare. The main reason, on the one hand, is lacking suitable multiphase methods meanwhile existing models is complex to realize and has bad effects. On the other hand, multiphase interfaces are difficult to track and exist overlaps or gaps. In order to track multiphase interfaces simply and effectively using particle-based framework, we present a surface reconstruction strategy for multiphase interfaces that eliminating the overlaps and gaps.

II. RELATED WORK

For the past few years, with the application of SPH framework in fluid simulation, researchers carried out an in-depth study and exploration on reconstructing surfaces based on mesh-free method. Blinn presented the classic blobby spheres approach[1]. A new kind of implicit surface equations are employed using distance of sampling points to the scattered point as a parameter. The values of equations are used to determine whether the sampling points are on the surface, it successfully realize surface extraction of the discrete points. One of the defects is that high or low densities of particles would cause bumps or indentations on the surface. Muller et al.[2] proposed a reconstructing approach similar to level set method by calculating the color field function. Fluid's free surface can be constructed simply and rapidly by this method, but the surface extracted is rough and particles near the surface cause bumps. Zhu and Bridson modified Blinn's algorithm to correct local particle's density variations[3]. Primarily, they calculate fluid particles' weighted average of coordinates and radius on the basis of neighbor particles' position and radius. Then they apply particles' weighted average coordinates and radius to the calculation of reconstructing surfaces, thus they get a relatively smooth fluid surface with no obvious bumps. Adams et al.[4] put forward improved method by tracking the particle-to-surface distances over time which made the surfaces more smooth. Due to the surface expression using implicit function, it is time-consuming. Bhatacharya et al. [5]proposed a Level Set method that consider surface reconstruction as a optimization problem. The initial surfaces are smoothed through the iteration process to achieve smooth surface effects. Yu et al. [6] proposed a new reconstruction method for fluid surfaces. They only construct surfaces at the beginning of the simulation, and the next each frame they merely process triangular mesh segmentation and integration of operations to previous frame surface, no longer reconstruct fluid surfaces each frame. Akinci et al. [7] presented a parallel surface reconstruction and optimization algorithm, and addressed a surface tectonic line method[8]. Yu and Turk[9] introduced anisotropic kernels to the implicit surface which extract more real and smooth fluid surfaces.

In addition, SPH method's core thought is using summation of neighbor particles' value in local area to replace field function and its derivative. The distribution of neighbor particles at local area directly affect the calculation results and the accuracy in consequence. Particles on the surface have less neighbors than in the internal that cause smaller density computed by SPH formula. What's worse, it results in a series of influence and eventually leads to the surface distortion. Noting this problem, Schechter and Bridson[10]addressed Ghost SPH to compensate for surface particle's density variations. They create a random Ghost particle layer in the external of

original particle boundary. It modifies SPH kernel function by increasing particle number within the scope of support domain. Besides, Liu et al. [11]presented ASPH method to solve the SPH calculation problem of lacking neighbors in local region. They construct the distance covariance matrix of particles and its neighbors to analyze the distribution characteristics of the surrounding particles. Then they acquire the eigenvectors served as distance weight of different direction to achieve the anisotropic effects. In order to eliminate this problem when using SPH method simulate gas, Ren et al. [12] proposed a compensation method for calculation of density and forces, which only simulates visible particles and does not generate surrounding particles in other area. Yu and Turk[9]inspired by ASPH method and introduced the anisotropic kernels to surface reconstruction. In their method, each particle own a unique kernel function obtained by the distribution of the particle's neighbors. It can better deal with planar fluid even thin surfaces with sharp features can still obtain a good effect.

Due to the complex movement and topology changes of multiphase interface, tracking the multiphase interfaces is a very challenging problem which also determines the effects of multiphase fluids simulation. In recent years, researchers put forward a lot of methods for multiphase interface tracking, mainly including the volume of fluid method (VOF) [13], moving grids and Lagrange grid method [14][15], level set method[16][17], particle-based surface reconstruction method[18][19]. However, most methods can not be directly applied to particle-based fluid simulation, and exist methods are complex to implement, as well as the fluid surface and interface effects reconstructed are rough with vacuums or overlaps. Therefore, we propose an simple method to construct the multiphase interfaces using anisotropic kernels, which can remove the overlaps and voids of the multiphase interfaces in surface reconstruction.

III. SPH FLUID FRAMEWORK

In the Lagrangian description, flow controlled partial differential equations of Navier-Stokes for fluids can be expressed as

$$\frac{d\rho_i}{dt} = -\rho_i \nabla \bullet \mathbf{v}_i , \tag{1}$$

$$\rho_i \frac{D\mathbf{v}_i}{Dt} = -\nabla p_i + \rho_i g + \mu \nabla^2 \mathbf{v}_i \ ; \tag{2}$$

Where $\mathbf{v}_i$ is the velocity, ρ_i is the density, p_i is the pressure, μ is the viscosity coefficient and g represents the external force field. Eq.(1) is mass conservation equation and Eq.(2) momentum conservation equation.

The theory of SPH is to utilize the form of discrete particles to characterize the successive fields and use integration to approximate the fields. For particle i at location $\boldsymbol{x}_i$,

$$\langle A(\boldsymbol{x}_i)\rangle = \sum_j m_j \frac{A_j}{\rho_j} W(\boldsymbol{x}_i - \boldsymbol{x}_j, h) \tag{3}$$

Where m_j and ρ_j represent particle mass and density respectively, $W(\boldsymbol{x}_i - \boldsymbol{x}_j, h)$ is the smoothing kernel and h the smoothing radius.

Applying Eq.(3) to the density of a particle i at location $\boldsymbol{x}_i$ yields the summation of density

$$\rho_i = \sum_j m_j W(\boldsymbol{x}_i - \boldsymbol{x}_j, h) \tag{4}$$

Therefore forces between particles including pressure $\mathbf{f}_i^P$ and viscous force $\mathbf{f}_i^{\nu}$ can be represented as

$$\mathbf{f}_i^P = -\sum_j m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2}\right) \nabla W_{ij} \tag{5}$$

$$\mathbf{f}_i^{\nu} = \mu \sum_j m_j \frac{\mathbf{v}_{ji}}{\rho_j} \nabla^2 W_{ij} \tag{6}$$

We employ Tait equation[[20]] to calculate the pressure, that is $p_i = \frac{\rho_0 c_S^2}{\gamma}\left(\left(\frac{\rho_i}{\rho_0}\right)^{\gamma} - 1\right)$. Where ρ_0 = 1000 is the rest density of the fluid, γ =7 is stiffness parameters and c_S is velocity of sound. For multiphase fluids modeling, we use Solenthaler's[19] approach.

IV. SURFACE RECONSTRUCTION USING ANISOTROPIC KERNELS

Normally, the color field definition [2] as follows

$$\phi(\boldsymbol{x}) = \sum_j \frac{m_j}{\rho_j} W(\boldsymbol{x} - \boldsymbol{x}_j, h_j) \tag{7}$$

In eq.(7), W is isotropic kernel function and its expression is

$$W(\mathbf{r}, h) = \frac{\sigma}{h^d} P\left(\frac{\| \mathbf{r} \|}{h}\right) \tag{8}$$

where σ is scaling factor, d is dimension of simulation, $\mathbf{r}$ is radial vector, and P is symmetric decaying spline with finite support.

To cope with the problem of density distributions near the surface, anisotropic kernel method[9] apply one step of diffusion smoothing to the location of the kernel center $\boldsymbol{x}_i$, which has denoising effect. The updated kernel center $\bar{\boldsymbol{x}}_j$ can be written as

$$\bar{\boldsymbol{x}}_i = (1-\lambda)\boldsymbol{x}_i + \lambda \sum_j w_{ij} \boldsymbol{x}_j \Big/ \sum_j w_{ij} \tag{9}$$

Where w is weighting function and $0 < \lambda < 1$.

Anisotropic kernel method[[9]] captures the density distribution more accurately by replacing h with a $d \times d$ real positive definite matrix G, it redefines W to be an anisotropic kernel

$$W(\mathbf{r}, \mathbf{G}) = \sigma \det(\mathbf{G}) P\left(\|\mathbf{G}\mathbf{r}\|\right) \tag{10}$$

where $\mathbf{r} = \boldsymbol{x} - \bar{\boldsymbol{x}}_j$, $\boldsymbol{x}$ can be treated as any position, the matrix $\mathbf{G}$ rotates and stretches the radial vector $\mathbf{r}$.

Then, the anisotropic kernel method applies weighting principal component analysis (WPCA) to determine $\mathbf{G}$. WPCA begins by calculating a weighting mean of data points. Afterwards, it constructs a weighting covariance matrix $\boldsymbol{C}$ and performs an eigendecomposition on $\boldsymbol{C}$. The resulting eigenvectors provide the principal axes. At last, it construct an anisotropy matrix $\mathbf{G}$ to match W with the output of WPCA.

The covariance matrix and weighting mean of particle i used by anisotropic kernel method are as follows

$$\boldsymbol{C}_i = \sum_j w_{ij} (\boldsymbol{x}_j - \bar{\boldsymbol{x}}_i)(\boldsymbol{x}_j - \bar{\boldsymbol{x}}_i)^{\mathrm{T}} \Big/ \sum_j w_{ij} \tag{11}$$

$$\bar{\mathbf{x}}_i = \sum_j w_{ij} \mathbf{x}_j \Big/ \sum_j w_{ij} \tag{12}$$

The function w_{ij} is an isotropic weighting function

$$w_{ij} = \begin{cases} 1 - \left(\dfrac{\|\mathbf{x}_i - \mathbf{x}_j\|}{l_i} \right)^3 & \|\mathbf{x}_i - \mathbf{x}_j\| < l_i \\ 0 & \text{other} \end{cases} \tag{13}$$

Where l_i is radius of neighbor scope. In order to include enough neighbor particles and obtain reasonable anisotropic information, we choose $l_i = 2h_i$.

For each particle, Singular value decomposition(SVD) can be performed on covariance matrix $\boldsymbol{C}_i$ that is

$$\boldsymbol{C}_i = \boldsymbol{R}\boldsymbol{\Sigma}\boldsymbol{R}^{\mathrm{T}} \tag{14}$$

Where $\boldsymbol{R}$ is a rotation matrix with principal axes as column vectors, $\boldsymbol{\Sigma} = diag(\sigma_1, \cdots, \sigma_d)$.

In the above formulas, $\boldsymbol{R}$ is a 3×3 rotation matrix with the eigenvectors of $\boldsymbol{C}_i$ as column vectors. Each column $\boldsymbol{R}_i$ represents distribution axis of $\boldsymbol{C}_i$ which corresponding to eigenvalue σ_i in $\boldsymbol{\Sigma}$. $\boldsymbol{\Sigma}$ is a diagonal matrix with eigenvalues $\sigma_1 \geq \cdots \sigma_d$, so the quantity of neighbor particles is large in the $\boldsymbol{R}_1$ direction and smallest in the $\boldsymbol{R}_3$ axis.

To avoid extreme conditions and unexpected situation occurs, anisotropic kernel method modifies matrix $\boldsymbol{C}_i$. Primarily, it controls σ_i, if $\sigma_1 / \sigma_d \geq k_r$, then σ_i is replaced by σ_1 / k_r. Next, using $G = k_n I$ replaces anisotropic kernels for internal fluid particles as well as isolated particles. The modified $\boldsymbol{C}_i$ can be written as

$$\tilde{\boldsymbol{C}}_i = \boldsymbol{R}\tilde{\boldsymbol{\Sigma}}\boldsymbol{R}^{\mathrm{T}} \tag{15}$$

$$\tilde{\boldsymbol{\Sigma}} = \begin{cases} k_s diag(\sigma_1, \tilde{\sigma}_1 \cdots, \tilde{\sigma}_d) & N > N_t \\ k_n \mathbf{I} & \text{other} \end{cases} \tag{16}$$

Where $\tilde{\sigma}_i = \max(\sigma_i, \sigma_1 / k_r)$, N is the number of neighbor particles, N_t is threshold. Anisotropic kernel method employ $k_r = 4$, $k_s = 1400$ to assure $\|k_s \mathbf{C}\| \approx 1$, and $k_n = 0.5$, $N_t = 25$.

To make kernel W transform according to $\tilde{\boldsymbol{C}}_i$ and keep the original form, $\boldsymbol{G}$ is expressed as

$$\boldsymbol{G} = \frac{1}{h} \boldsymbol{R}\tilde{\boldsymbol{\Sigma}}^{-1}\boldsymbol{R}^{\mathrm{T}} \tag{17}$$

V. ANISOTROPIC SURFACE RECONSTRUCTION FOR MULTIPHASE INTERFACES

A. *Anisotropic kernel for multiphase Interfaces*

In essence, anisotropic surface reconstruction method is a kind of level set method. In order to construct the multiphase interfaces, level set methods usually use different level set function for each phase fluid, which is known as multiple level set method, and then construct surfaces of different fluids according to the different level set function. But directly using multiple level set methods, it tends to cause problems in the interface area. When adopting the isotropic method[[2]], the overlapping phenomena arise in multiphase interfaces. While using anisotropic surface construction algorithm appears a more serious overlapping phenomenon or crack problems at the interface.

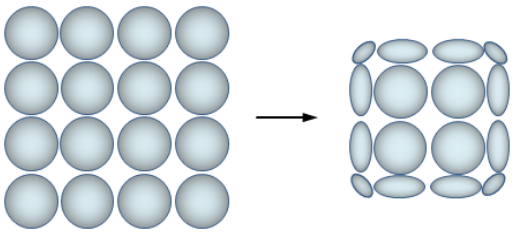


Fig. 1. Isotropic kernel transform to anisotropic kernel

Figure1 shows an example of the smooth kernel representation of an isotropic kernel function transformed into an anisotropic kernel function. Where the sphere and the

ellipsoid are the kernel function scope of particles, the left graph illustrates the isotropic kernel and the right graph shows the anisotropic kernel. It can be seen that after anisotropic transformation, the smooth kernel near the fluids surface changes from sphere to ellipsoid and the smooth kernel of internal particles remains spherical. Therefore, multiphase interfaces will show similar effects when employing anisotropic surface reconstruction method with a multiple level set, as shown in Figure 2. Where the left is the isotropic kernel method, and the right is anisotropic kernel method. It is obviously that the particles at the interface lack neighbor particles. With the purpose of including more neighbor particles, the anisotropic kernel method changes the smooth kernel, so the kernels of particles at the interface are transformed into ellipsoid. However, on account of using multiple level set, the fluid surfaces reconstructed have gaps at multiphase interfaces for no consideration of other phase particles' influence, which will seriously affect off-line rendering later.

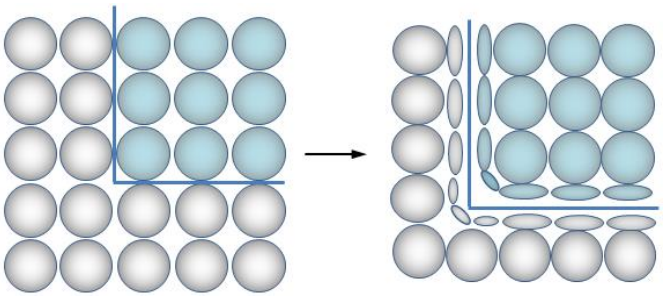


Fig. 2. Isotropic kernel transform to anisotropic kernel for multiphase interface

In addition, in order to remove noises and dispose uneven distribution of particles, anisotropic kernel method employ Laplacian smoothing for the location of kernel center(as shown in Eq.(9)). This process improves the uneven distribution of surface particles. Nevertheless, since the neighbors of surface particles are mainly in the inner side, the whole fluids shrink inwardly and the fluid volume is compressed slightly. It is not difficult to find that this smoothing is also a cause for the gaps of multiphase interface.

Moreover, when constructing fluid surface of each phase respectively with multiple level set method, it will generate overlap phenomena at multiphase interface, which is the common problem of level set method. As shown in figure 3, the fluid interface appears overlap phenomenon in the two-phase surface reconstruction. Where the left graph represents the ideal fluid surface of particle-based simulation, the graph in the middle expresses the fluid surfaces reconstructed using multiple level set, and the right graph shows the entire surface of the fluid domain. Because of the color field method[2]is a kind of level set method, we take color field method as an example to analyze the reason of multiphase interface's overlapping issue. As shown in figure 3, it assumes that the color field value of blue fluid is 10 and the color field value of green fluid is 20. The color field values of blue fluid particles near the surface are linearly reduced from 10 to 0, while the color field values of green particles near the surface are decreased from 20 to 0. At this time, if we select a uniform color field value to construct the surface of the entire fluid domain, there will emerge overlaps at the two-phase interface. This is because the color field values of the two fluids near the surface have overlapping areas (one from 10 to 0, the other from 20 to 0). In consequence, the two-phase fluid interfaces intersect when choosing a uniform color field value to construct the entire fluid surface. For example choosing the color field 0 to reconstruct the whole surface of fluid domain that exhibited in the right graph of figure 3(There are still overlapping issues with other values).

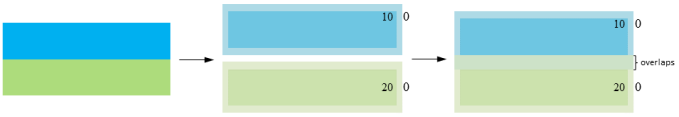


Fig. 3. Schematic diagram of two-phase interface overlapping case

Consequently, in order to eliminate the defects of surface reconstruction in multiphase fluids simulation, we consider the contribution of other phase fluids when tracking fluid surface for one phase, and carry out some special treatment.

Primarily, since the kernel centers Eq.(9) used by anisotropic surface reconstruction method does not take into account the influence of other phases, we modify the Eq.(9) as follows.

$$\bar{\boldsymbol{x}}_i' = (1-\lambda)\,\boldsymbol{x}_i + \lambda \sum_{k}^{n}\sum_{j} w_{ikj}\boldsymbol{x}_{kj} \Big/ \sum_{j}^{n}\sum_{j} w_{ikj} \tag{18}$$

Where w is the weight function, λ is a constant that between 0 and 1, n represents the number of fluid phases.

Similarly, we alter the covariance matrix $\boldsymbol{C}_i$ of Eq. (11) as below.

$$\boldsymbol{C}_i' = \sum_{k}^{n}\sum_{j} w_{ikj}(\boldsymbol{x}_j - \bar{\boldsymbol{x}}_i)(\boldsymbol{x}_j - \bar{\boldsymbol{x}}_i)^{\mathrm{T}} \Big/ \sum_{k}^{n}\sum_{j} w_{ikj} \tag{19}$$

In the above formula, w_{ikj} is weight function that is created taking into account the distance between particle i and particle j. The expression is as follows.

$$w_{ikj} = \begin{cases} 1-\left(\dfrac{\left\|\mathbf{x}_i - \mathbf{x}_j\right\|}{l_i}\right)^3 & \left\|\mathbf{x}_i - \mathbf{x}_j\right\| < l_i \\ 0 & \text{other} \end{cases} \tag{20}$$

Where l_i is support domain radius, k indicates the type of fluid which has no effect on the weight function, so w_{ikj} is the same as Eq.(13).

According to the covariance matrix of Eq.(19), similar to the matrix $\boldsymbol{G}$, we produce matrix $\boldsymbol{G}'$ as below.

$$\boldsymbol{G}' = \frac{1}{h}\boldsymbol{R}'\tilde{\boldsymbol{\Sigma}}'^{-1}\boldsymbol{R}'^{\mathrm{T}} \tag{21}$$

In general, the color field used for extracting surface of each phase in multiphase simulation can be expressed as:

$$\phi'(\boldsymbol{x}) = \sum_j \frac{m_j}{\rho_j} W(\boldsymbol{x} - \boldsymbol{x}_j, \boldsymbol{G}'_j) \tag{22}$$

B. Surface reconstruction stragegy

In the example of figure 3, the color field used is an unsigned distance field. The unsigned color field is usually employed for surface reconstruction, while we exploit signed color field with the purpose of preferably extracting of multiphase interface. Besides, We adopt strategy "binary tree" for surface reconstruction with multiphase interface, as shown in figure4.

We illustrate our reconstruction strategy in a four-phase fluid simulation, as shown in figure 4. In the figure, nodes 3, 5, 6 and 7 indicate four different fluids, and nodes 1, 2 and 4 represent fluid surface of four-phase, three-phase and two-phase respectively. When constructing the fluid surface, we extract only one phase from the entire fluid domain at each iteration. The reason for this is that when expressing the fluid surface, we can use the symmetrical color field with positive and negative values (signed color field) to distinguish the two-phase interface. The adoption of signed color field make the color field values between two-phase interfaces transform uniformly from negative to positive. For example, in order to extract the surface of fluid 1 (node 3) and the surface of the other three-phase (node 2) from the four-phase fluids (node 1), we assign color field to $\phi'(x) = 1$ and $\phi''(x) = -\phi'(x)$, respectively. In this way, two kinds of fields can be interpolated according to Eq.(22), one represents the fluid domain of fluid 1 and another denotes the fluid domain of the other three-phase. Then we can reconstruct fluid surfaces of fluid 1 and the other three-phase according to the interpolated two kinds of field values. While the selection of field values for surface reconstruction needs pay much attention, we choose $\varepsilon\phi'(x)$, $-\varepsilon\phi'(x)$ ($0 < \varepsilon < 0.1$, we adopt $\varepsilon = 0.01$) to construct the surfaces. This ensures the reconstructed fluid surfaces is approximately one face at multiphase interface without serious vacuums and overlaps. Next, we extract the surface of fluid 2 and the other two-phase from the remaining three-phase fluid in the same manner. Finally, we repeat the above ways to find the surface of fluid 3 and fluid 4. In summary, we can separately extract fluid surface for each phase employing a "binary tree" reconstruction strategy, which can avoid vacuums and overlaps between multiphase interfaces. It is apparently that this reconstruction strategy can be used to extract fluid surface of arbitrary phases in a similar way meanwhile maintain a good effect of multiphase interface.

In a word, the surface reconstruction strategy for multiphase fluid (n-phase) proposed in this paper can be summarized as follows:

Firstly, assign color field $\phi'(x)$, $-\phi'(x)$ to particles of one phase fluid and the other n-1 phase fluid, respectively;

Secondly, interpolate the signed color field to the one phase fluid and the other n-1 phase fluid and choose $\varepsilon\phi'(x)$, $-\varepsilon\phi'(x)$ as the surface field value separately.

Thirdly, reconstruct the surface for the one phase fluid according to the chosen surface field value;

Finally, repeat the above steps until the surface of each phase is reconstructed.

The addressed surface reconstruction strategy for multiphase interface has several merits. For one thing, the color field method commonly used compel the color field values linearly reduced from $\phi(x)$ to 0 approximately near the fluid surface, while using signed color field can make color field values linearly decreased from positive to 0, and further weakened to negative at two-phase interface. So, it ensures the surface reconstructed uniform at two-phase interface. In a word, the interface area is averagely divided biases are avoid. Otherwise, it maybe have vacuums or overlaps. For another thing, choosing a relatively small surface field value $\varepsilon\phi'(x)$, $-\varepsilon\phi'(x)$ guarantee that no extra area be reconstructed. It is effective to avoid reconstructing domain that is not the fluid surface while whose color field values are almost the same as fluid surface. Moreover, because $\varepsilon\phi'(x)$ is small enough, the multiphase interface can approximately regard as one interface without overlaps.

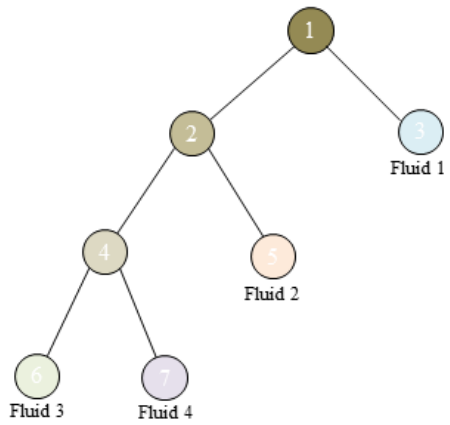


Fig. 4. Schematic diagram of surface reconstruction strategy for multiphase interface

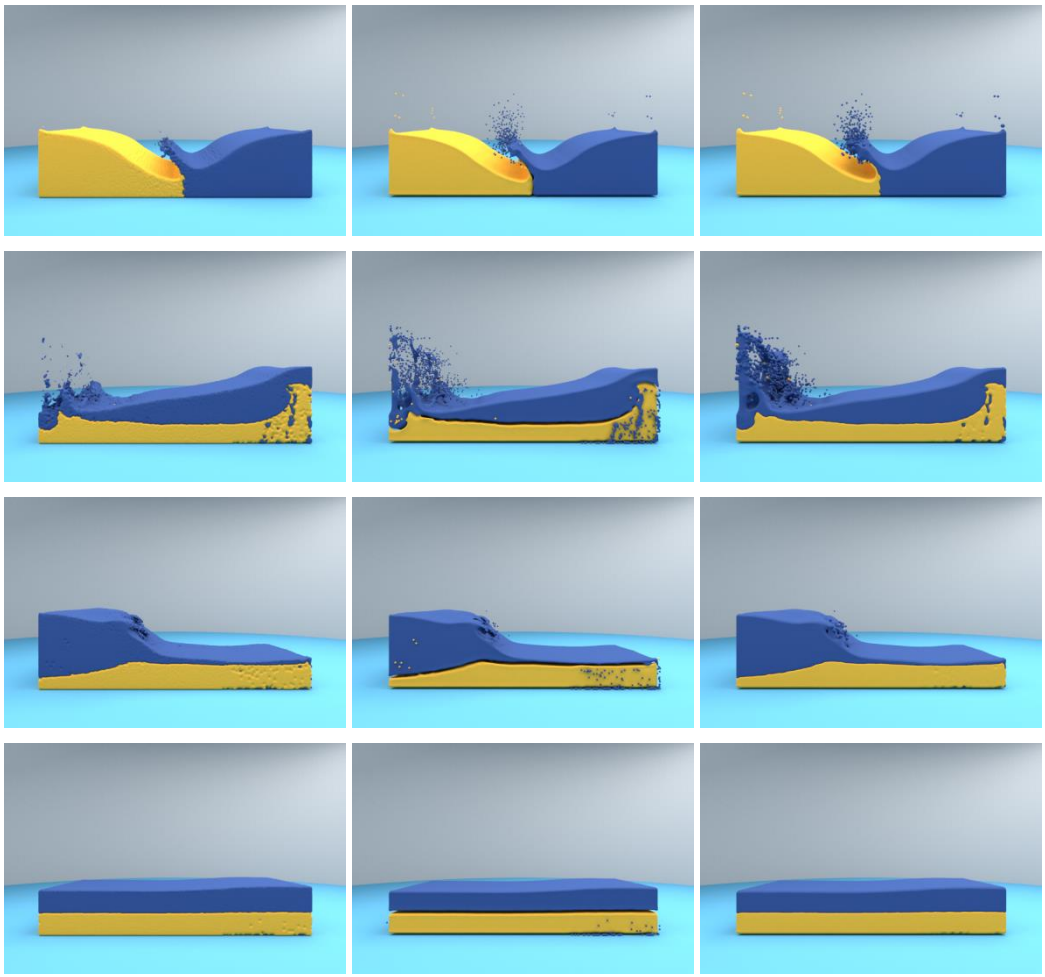
VI. IMPLEMENT AND RESULTS

We implement several experiments to demonstrate the validity of surface reconstruction method for multiphase interface presented. The experiments is performed on a graphic workstation with Intel Xeon E5-2637 v2(15M Cache, 3.50 GHz @ 4 Cores), 80 GB memory, NVIDIA Quadro K4000 GPU. The simulation is actualized with C++ language and OpenMP is served for parallelization. Neighbor Search is implemented by spatial Hash method with a width h uniform space background mesh. We employ surface reconstruction

method for multiphase interface presented in this paper with Marching Cubes algorithm to extract fluid surfaces. The real time simulation and rendering is achieved using OpenGL 3D Graphics Library, and offline high quality rendering is implemented with Blender's ray tracing engine-Cycles.

TABLE I. THE SETTING AND STATISTICS OF TWO-PHASE BREAKING DAM SIMULATION

Item	Value
Simulation domain size	$24m \times 24m \times 24m$
Smoothing kernel function	Cubic splines
Blue particles	126k
Yellow particles	126k
Density of blue fluid	$200\ kg/m^3$
Density of yellow fluid	$1000\ kg/m^3$
Smoothing radius	0.2m
Diameter of fluid particle	0.1m



(a) Isotropic kernel method (b) Anisotropic kernel method (c) Our method

Fig. 5. Two-phase breaking dam experiment

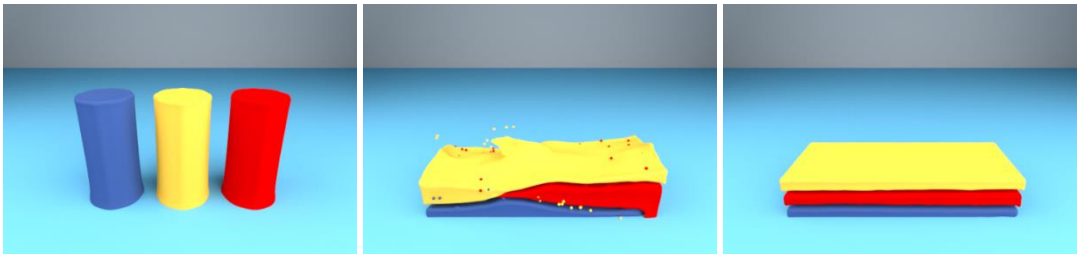
Figure 5 illustrates surface reconstruction results of isotropic kernel method [2], anisotropic kernel method [9]and our method in two-phase breaking dam experiment. The setting of this experiment is shown in Table 1. The process of two-phase breaking dam is that two cuboids of different phase are fallen by gravity at the beginning of the simulation, and then two-phase fluids are gradually contacted and mixed. Afterwards, under the action of pressure and interface force as well as other forces, the fluid of smaller density will be "floating" in the movement, while the fluid of larger density will gradually "sinking". Eventually, the fluids' movement tends to finish and fluids are divided into two layers with a clear interface. As shown in figure 5(a), it is observed that the multiphase fluid surface reconstructed using isotropic kernel method is thoroughly rough and has overlaps and vacuums at two-phase interface, meanwhile the surface effects is poor after rendering. Figure 5(b) shows the fluid surface reconstructed by anisotropic kernel method, which is more smooth than isotropic kernel method. But there has distinct vacuum at the two-phase interface, which seriously affects the realistic effects of multiphase fluids simulation. From figure 5(c) we can find that the fluid surface reconstructed with our method is

comparatively smooth and has no gaps or overlaps at two-phase interface. It exhibits a better realistic effect and proves that our method can eidetically simulate two-phase fluids interaction.

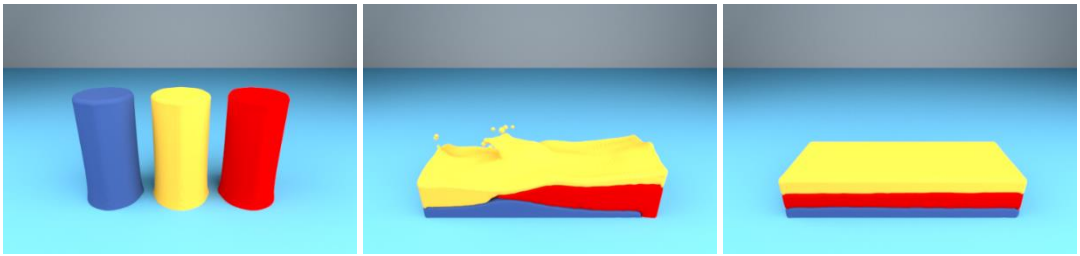
TABLE II. THE SETTING AND STATISTICS OF THREE-PHASE BREAKING DAM SIMULATION

Item	Value
Simulation domain size	$24\text{m} \times 24\text{m} \times 24\text{m}$
Smoothing kernel function	Cubic splines
Blue particles	13325
Yellow particles	13325
Red particles	13325
Density of yellow fluid	$100\,kg/m^3$
Density of red fluid	$300\,kg/m^3$
Density of blue fluid	$900\,kg/m^3$
Smoothing radius	0.2m
Diameter of fluid particle	0.1m

Figure 6 exhibits surface reconstruction results of anisotropic kernel method [9]and our method in three-phase cylinder fluid breaking dam experiment. The setting and statistics of this experiment is listed in Table 2. The process of three-phase cylinder fluid breaking dam experiment is as follows. Firstly, When simulation starts, three-phase fluids are fallen due to gravity and gradually begin to contact and mix. Then, similar to two-phase breaking dam experiment, smaller density fluid floats with the movement, while larger density fluid gradually sinks. Because yellow fluid's density is the smallest, it flows upward due to squeeze by the other two-phase fluids. Next, three-phase fluids intensely collide and fully mix. Finally, the fluids' movement tends to finish and fluids are divided into three layers with clear interfaces. As shown in figure 6(a), the fluid surface reconstructed by anisotropic kernel method still has obvious gaps at three-phase interface, which resulting in poor multiphase interface effects. While figure 6(b) illustrates the results of our method in three-phase flow simulation with smooth and flat surface effects, which has no vacuums or overlap and further demonstrate the validity of our method.



(a) Anisotropic kernel method



(b) Our method

Fig. 6. Three-phase breaking dam experiment

CONCLUSION

With regard to the problem of overlaps and vacuums at multiphase interfaces when reconstructing multiphase fluids surface using anisotropic methods with multiple level set, we propose a simple and effective surface reconstruction strategy for multiphase interfaces. It modifies the calculation of anisotropic kernel function and adopts a "binary tree" strategy to reconstruct fluid surface. The experimental results demonstrate that our method can simulate the interaction of multiphase fluids accurately and remove the gaps and overlaps in multiphase interface. Overall, our method is effective for particle-based multiphase fluids simulation and can be applied to other relevant applications. Future work would be extending the proposed approach to large scale scenarios and multiphase mixture simulation.

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