Viscosity-based Vorticity Correction for Turbulent SPH Fluids

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ABSTRACT

A critical problem of Smooth Particle Hydrodynamics (SPH) methods is the numerical dissipation in viscosity computation. This leads to unrealistic results where high frequency details, like turbulence, are smoothed out. To address this issue, we introduce a viscosity-based vorticity correction scheme for SPH fluids, without complex time integration or limited time steps. In our method, the energy difference in viscosity computation is used to correct the vorticity field. Instead of solving Biot-Savart integrals, we adopt stream function, which is easier to solve and more efficient, to recover the velocity field from the vorticity difference. Our method can increase the existing vortex significantly and generate additional turbulence at potential position. Moreover, it is simple to implement and can be easily integrated with other SPH methods.

Keywords: Smooth Particle Hydrodynamics, turbulence, Lagrangian vortex method, fluid simulation.

Index Terms: I.3.7: Three-Dimensional Graphics and Realism; H.5.1: Multimedia Information Systems-Artificial, Augmented, and Virtual Realities;

1 INTRODUCTION

Particle-based fluid simulation is a hot issue in computer graphics which has a huge research and application demands. SPH approach, as a Lagrangian method, is able to represent the splash of water vividly. Recently, several solutions have been published to enforce incompressibility to obtain realistic results [4]. However, numerical dissipation is still not solved completely, which lead to the smoothing out of some interesting details.

Vorticity confinement (VC) is a classical method to enhance vortex by adding new forcing term, which has been successfully used in grid-based smoke [3] and SPH fluids [5]. However, it tends to add more energy than numerical dissipation, and only existing vortices can be amplified. Lagrangian vortex methods use particles [6], sheets [1] and filaments [7] to simulate turbulent fluids and vortex details around moving objects [9]. Bender [2] introduced the micropolar fluid model for inviscid fluid, which could capture the microrotation of fluid particles. But the equivalent of three Poisson solvers is still needed to obtain velocity from vorticity, which are thoroughly time-consuming. Zhang [8] solve this issue in advection-projection scheme, which cheaply captures much of the lost details for smoke. However, this method does not work well in liquid scenes.

In SPH-based methods, viscous force in N-S equations cannot be directly discretized for fluid simulation, so artificial viscosities are widely used to achieve plausible results. The numerical dissipation during viscous computation result in losses of vortexes and turbulence details. However, only a few papers have addressed this particular type of numerical dissipation.

To solve the above-mentioned problems, we introduce a viscosity-based vorticity correction scheme, correcting the vorticity field by the energy dissipated rate during the viscosity computation. With the help of stream function, we are able to refine the velocity using a reasonable augmentation of curl field, which can restore a vivid yet controllable vortex and turbulence effect. Our method can not only increase the existing vortex, but also generate turbulence at potential locations of new vortexes. Besides, the solution of the rotation field and the correction of the velocity are added in the prediction-correction scheme after applying viscous force. So, it could be integrated with prediction-correction scheme easily, without affecting the subsequent pressure correction step.

2 THEORY

In our algorithm, each particle has a vorticity which is expressed by the rotation of the velocity field $\boldsymbol{\zeta} = \nabla \times \boldsymbol{v}$. In particle system, it is discretized into the following form:

$$\boldsymbol{\zeta}_i = \nabla \times \boldsymbol{v}_i = -\frac{1}{\rho_i}\sum_j m_j(\boldsymbol{v}_i - \boldsymbol{v}_j) \times \nabla_i W_{ij} \tag{1}$$

During the flow, energy will be transferred among the particles. When the velocity of a particle changes, its vorticity also changes and affects the trajectory of the particle. In this process, if the specific value of the vorticity be calculated or if its feedback is not applied to the velocity, some important details will be lost.

We derive a $\tilde{\boldsymbol{v}}_i^{vis}$, which is velocity after applying viscosity force, for each fluid particle from the previous time step $\boldsymbol{v}_i^n$: $\tilde{\boldsymbol{v}}_i^{vis} = \boldsymbol{v}_i^n + \boldsymbol{a}_i^{vis} \cdot \Delta t$. If $\tilde{\boldsymbol{v}}_i^{vis}$ is greater than $\boldsymbol{v}_i^n$, which means particle is boosted by another one through viscosity, we refine its vorticity field as well:

$$\delta\boldsymbol{\zeta}_i = \boldsymbol{\zeta}_i \cdot \sqrt{R_i}\alpha \tag{2}$$

Where is R_i the energy change rate caused by viscosity:

$$R_i = \frac{\delta E_i}{E_i} = \frac{\frac{1}{2}m_i\tilde{\boldsymbol{v}}_i^{vis^2} - \frac{1}{2}m_i\boldsymbol{v}_i^{n^2}}{\frac{1}{2}m_i\boldsymbol{v}_i^{n^2}} = \frac{\tilde{\boldsymbol{v}}_i^{vis^2} - \boldsymbol{v}_i^{n^2}}{\boldsymbol{v}_i^{n^2}} \tag{3}$$

and α is the enhancement parameter range from 0 to 1.

As shown in Figure 1, some of the kinetic energy could be represented by the angular velocity. After the change rate of the kinetic energy R_i of certain particle is obtained, if the energy represented by the angular velocity is expanded by the same ratio,

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its value is still within a reasonable range. In physics, the vorticity has a certain relationship with the angular velocity which is at most equal to twice the angular velocity. So, the $\sqrt{R_i}$ in formula 3 make sense.

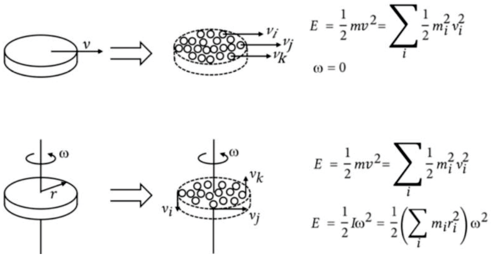

Figure 1: The energy carried by the angular velocity is always less than or equal to the kinetic energy.

To recover the velocity field from vorticity, we apply discretized stream function as a bridge for two fields.

$$\boldsymbol{\Psi}_i = \sum_j \frac{\delta\zeta_j V_j}{4\pi r_{ij}} \tag{4}$$

where V_i is the volume of each particle. The velocity correction of the particle at the next moment could be expressed as: $\boldsymbol{\delta v} = \nabla \times \boldsymbol{\Psi}$.

Combined the velocity correction and the velocity after advection to obtain the corrected velocity. Note that all of these processes run in the advection scheme which naturally guarantees incompressibility.

3 EXPERIMENT AND DISCUSSION

To exhibit the effectiveness of our approach, the following experiment are carried out. The enhancement parameters in all these experiments is 0.06.

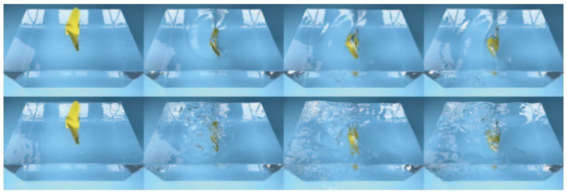

Figure 2: Propeller spinning water with 1.59 M fluid particles. IISPH method (upper row): neither the strong flow nor the complex flow has been preserved due to highly energy dissipation. Our method (lower row): two ideal vortexes were formed on each side of the propeller.

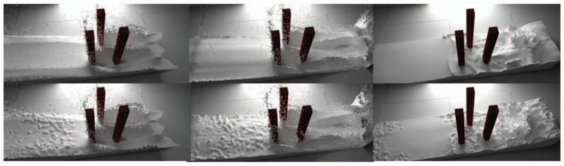

Figure 3: Liquid flowing through obstacles with 1.12 M fluid particles. IISPH (upper row): liquid just go around the pillars and forms a little splash. Our method (lower row): big splash was formed when break-dam hit the obstacles. After the water stream stops, rough turbulence and vortexes remains vibrant.

Figure 2 shows the results of classical IISPH and our method under strong agitation. It proves that our method could enrich the vortex detail realistically. This drastic flow with violent water runs out of the container, which could also prove the outstanding stability of our algorithm. In Figure 3, a classical dam-break test was carried out, which shows that our method also does well in forming turbulence. In Figure 4, a boat-sinking experiment is carried out to demonstrate the stability. The huge potential energy of the boat transforms into kinetic energy of the fluid particles. As the time goes by, water gradually calms down, which means no excessive energy is added to the system. Our method is stable enough to deal with some extreme conditions with less dissipation.

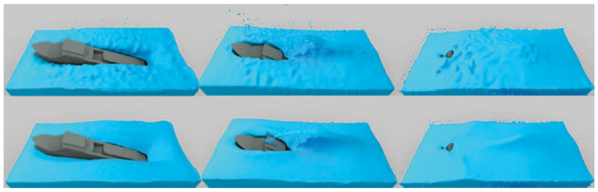

Figure 4: Boat sinking with 2.93 M fluid particles. IISPH (upper row): high-resolution turbulence and vortex details were smoothed out, only remains a little splash. Our method (lower row): rich turbulence and vortex details could be observed.

4 CONCLUSION

We present a particle-based turbulence and vortex method that deriving the vorticity difference from the viscosity energy dissipation to restore velocity field. This method can significantly reduce numerical dissipation, increase the existing vortex and generate new turbulence at potential locations. Experiment shows that the proposed method greatly enhances the turbulent effects compared to classical method.

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