Surface Tension Fluid Simulation with Adaptiving Time Steps

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Abstract. In this article, a surface tension fluid simulation algorithm based on IISPH is proposed. Based on the SPH algorithm, the surface tension and the adhesion model are constructed to solve the problem about particle clustering, fluid surface area minimization and interaction between different particles. The method can make the simulation effect of fluid be more in line with the actual physical scene. Furthermore, an adaptive time-stepping method is added in the algorithm. The efficiency of the simulation is significantly improved compared to the constant time-stepping.

Keywords: Surface tension · Cooperative visualization · Adaptiving time steps · Implicit Incompressible SPH

1 Introduction

Fluid phenomenon exists widely in our daily life, such as rain, oil and so on. As a visualization technique, the fluid simulation has important applications in the field of collaborative visualization, and also has been a great challenge at the same time. Fluid simulation methods can be broadly divided into two categories: Eulerian method and Lagrangian method. In the Lagrangian method, the Smoothed Particle Hydrodynamics (SPH) [1, 2] method is a very popular algorithm because of its simplicity. The early SPH used the EOS equation to directly calculate the pressure of the particles, which is called standard SPH (SSPH) [3, 4]. The SSPH has a good effect on the simulation of compressible fluids, but it will lead to a strong sense of compression in the visual. Later, Becker et al. used the Tait equation to replace the ideal gaseous equation and use a high hardness control coefficient, which is called Weakly Compressible SPH (WCSPH) [5]. The WCSPH significantly increases the authenticity of the simulation by limiting the time step, but reduces the efficiency of the algorithm. In order to improve the efficiency of the algorithm, Solenthaler and Pajarola proposed Predictive-Corrective Iteration SPH (PCISPH) [6]. PCISPH can set the global maximum density fluctuations, and use Predictive-Corrective Iteration to achieve fluid incompressibility, and the algorithm eliminates the limitation of time step in WCSPH, which improves the overall efficiency of the algorithm by 10–50 times [7]. There are other similar algorithms, such as the Local Poisson SPH [8] and the Position Based method [9]. The above methods are based on the state equation to calculate the pressure, there is another way to achieve fluid incompressibility by projection method, which is called Incompressible SPH

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(ISPH) [10–12]. The main idea of this method is to use the force outside the pressure to predict the middle speed of the particle, and then solves the pressure Poisson equation, and finally calculates the other properties according to the pressure, but this will make the calculation cost significantly improved. To solve this problem, Ihmsen et al. proposed the Implicit Incompressible SPH (IISPH) [13]. IISPH constructs a similar iterative algorithm by carefully constructing the pressure Poisson equation and solving the linear system by using the Relaxed Jacobi method. IISPH is better than PCISPH in algorithm stability, convergence speed. Cornelis et al. demonstrated the excellent properties of IISPH once again by combining IISPH and FLIP methods [14].

Surface tension is an important physical property of fluid phenomena, and its simulation research has been an important part of the fluid simulation. The surface tension is generated by the cohesion between adjacent fluid particles. Using SPH method to simulate the surface tension of the fluid is a very challenging problem. The density of the fluid particles at the fluid and air junctions is too small because of the lack of neighbors, and this causes a problem of particles clustering. In addition, there are many other problems, such as surface curvature minimization and momentum conservation. In 2005, Tartakovsky and Meakin [15] proposed a method by using molecular cohesion to produce fluid surface tension, which uses cosine functions to control the gravitational and repulsive forces between particles. Becker and Teschner [16] used the kernel function of the SPH method to replace the cosine function to control the computational range of the gravitational force. However, both of these methods can not effectively solve the problem of particle clustering. Later, Akinci et al. [17] proposed a method of constructing cohesion and implemented on PCISPH. This method can solve the above problems. However, when the surface tension or the attraction force acting on the fluid is the main force, the time step is limited.

We propose a surface tension fluid simulation algorithm based on implicit incompressible SPH method. The experimental results show that the surface tension and adsorption force model proposed in this paper can minimize the surface area of the fluid. Furthermore, the efficiency of the simulation is significantly improved compared to the constant time-stepping.

2 Modeling of Surface Tension and Adhesion

Similar to Akinci et al.'s method [17], our surface tension model effectively solves the problem of gravitational repulsion and the minimization of the fluid surface area.

First of all, the cohesion between the particles will be computed based on the size of the distance between the particles to create gravitational and repulsive. The cohesion between the particles will produce gravitational and repulsive forces according to the distance between the particles. Similar to the force between the molecules, when the distance is too large to produce gravity, the distance is too small to produce repulsion until the gravitational and repulsive balance. The form is as follows:

$$a_i^c = -\delta \sum_{\mathrm{j}} m_j (x_i - x_j)\, e(|x_j - x_i|) \tag{1}$$

where j is the neighbor particles of i, m denotes mass, x denotes the displacement of particles, and e is a spline function.

As can be seen from (1), the spline function e determines the nature of F, just like a kernel function. F should meet the following conditions: when the distance between the particles is less than a certain threshold, F is the repulsive force; when the distance between the particles is greater than the threshold, F produces gravitational force. It can be deduced that the spline function e is a piecewise function form. In this paper, we use the spline function proposed by Akinc et al. [17]:

$$e(r) = \frac{32}{\pi h^9} \begin{cases} (h-r)^3 r^3 & \frac{\mathrm{h}}{2} r \le h \\ 2(h-r)^3 r^3 - \frac{h^6}{64} & 0 < r \le \frac{h}{2} \\ 0 & otherwise \end{cases} \tag{2}$$

In order to better simulate the microscopic characteristics of the fluid surface, we also need an additional force to minimize the fluid surface area.

$$a_i^k = -\chi \sum_j \left(n_i - n_j\right) \tag{3}$$

where χ is the correction factor, and n is to avoid the display of the calculated surface curvature:

$$n_i = \mu \sum_j \frac{m_j}{\rho_j} \nabla W\left(|x_i - x_j|\right) \tag{4}$$

where μ is the scaling factor.

In summary, the complete surface tension can be expressed as:

$$a_i^{st} = \psi_{ij}\left(a_i^{\mathrm{c}} + a_i^k\right) \tag{5}$$

where $\psi_{ij} = \frac{2\rho_0}{\rho_i + \rho_j}$ is the control factor.

The adhesion is different from the surface tension, which is caused by the interaction between the different types of particles. The adhesion of this paper is mainly for the fluid-solid coupling problem, as following:

$$a_i^{ad} = -\gamma \sum_k \psi_{b_k}(x_i - x_k) g(|x_i - x_k|) \tag{6}$$

where γ is the adhesion coefficient, $\psi_{b_{\mathrm{k}}}$ is the volume of boundary particles, g is a spline function:

$$\mathrm{g}(r) = \frac{0.01}{h^5} \begin{cases} -\left(r - \frac{3h}{4}\right)^2 + \frac{h^2}{16} & \frac{h}{2} \wedge r \le h \\ 0 & otherwise \end{cases} \tag{7}$$

3 Adaptiving Time Steps

The upper bound of the time step of SPH numerical simulation is given by CFL (Courant-Friedrich-Levy). It can ensures that the velocity of the numerical propagation is faster than the velocity of the physical propagation, so that the numerical calculation is stable and convergent.

$$\Delta t_{CFL} \leq \lambda_v \left(\frac{h}{v_{max}} \right) \tag{8}$$

where $v_{max} = \max_i \|\boldsymbol{v}_i\|$ is the maximum of all fluid particles velocities, $\lambda_v < 1$ denotes safety factor.

In addition, the fluid simulation algorithm based on the SPH method also needs to consider the force of the fluid particle (the acceleration produced by the force):

$$\Delta t_f \leq \lambda_f \left(\frac{h}{f_{max}} \right) \tag{9}$$

where $f_{max} = \max_i \left\| \frac{d\boldsymbol{v}_i}{dt} \right\|$, $\lambda_f < 1$.

The final time step requires consideration of both of the above conditions:

$$\Delta t \leq \min\left(\Delta t_{CFL},\ \Delta t_f \right) \tag{10}$$

Algorithm 1. Adaptiving time steps for surface tension fluid simulation

while *animating* **do**
for *all particles i* **do**
    search neighbor particle *j*
for *all particles i* **do**
    compute $\rho(i)$, $p(i)$
    compute a_i^{st}, a_i^{ad}
for *all particles i* **do**
    compute the total acceleration a_i^{total}
compute time step by (10)
for *all particles i* **do**
    update $v_i(t+\Delta t) = v_i^* + \frac{\Delta t F_i^p(t)}{m_i}$
    update $x_i(t+\Delta t) = x_i(t) + \Delta t v_i(t+\Delta t)$
$t = t + \Delta t$

4 Experimental Results

In this section, we show the capabilities of our approach. Firstly we compare the simulation results with surface tension and without surface tension. Then, we discuss the improvement of computational efficiency by adaptiving time steps. All timings are given for an Intel 3.50 GHz CPU with 4 cores. The simulation software is parallelized with OpenMP.

4.1 Surface Tension

The following figures show the flow of a water-drop on the board. First of all, the water-drop acts as a free falling body. Then the water-drop moves around on the board when it touches the board. We use the IISPH method without surface tension and adhesion firstly. The fluid particles present a loose state, which is more severe at the edge of the fluid (Fig. 1). After rendering, the grain is still very strong in the edge of the fluid (Fig. 3), which is clearly inconsistent with the actual physical scene. After using the method in this paper, the fluid particles are not in a loose state and the effect of the fluid edge has been significantly improved (Fig. 2). After rendering, the fluid surface becomes smoother and ensures surface area be minimized (Fig. 4). The fluid becomes a slightly flat water-drop shape eventually (Table 1).

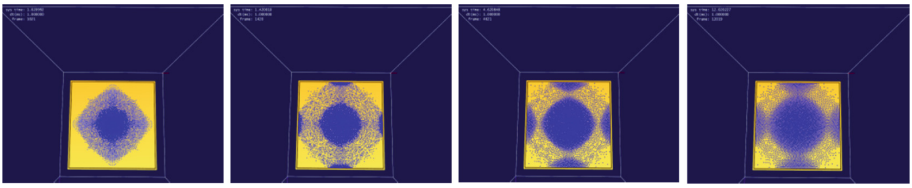

Fig. 1. Simulation without surface tension before rendering

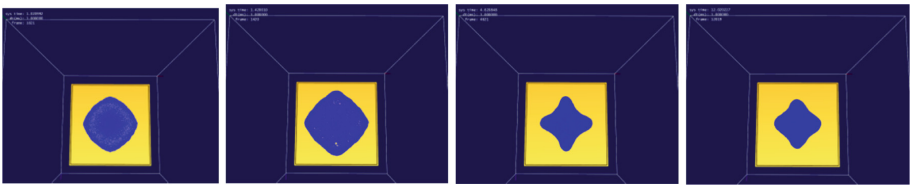

Fig. 2. Simulation with surface tension before rendering

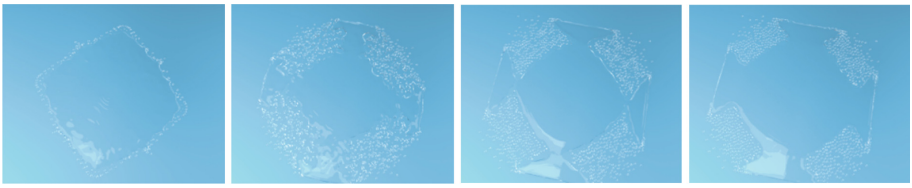

Fig. 3. Simulation without surface tension after rendering

Fig. 4. Simulation with surface tension after rendering

Table 1. The simulation parameters of experiment

Parameter	Value
The scale of simulation domain	8 m × 8 m × 8 m
The density of fluid particles	1000 kg/m^3
The smooth radii	0.2 m
The width of fluid particle	0.1 m

4.2 Adaptiving Time Steps

We designed a large 3D dam-break experiment to verify that our adaptive time-stepping algorithm improves the computational efficiency. The setting parameters of the experiment are shown in Table 2.

Table 2. The simulation parameters of 3D dam-break

Parameter	Value
The scale of simulation domain	12 m × 12 m × 8 m
The number of fluid particles	153 K
The number of boundary particles	73 K
The smooth radii	0.2 m
The width of fluid particle	0.1 m

The results of 3D dam-break experiment are shown in Table 3. In order to compare difference of the running time in different methods, we calculate the calculation time of the same experimental scenario in fixed time steps, and adaptiving time steps. Through the data in the Table 3, we can find that the adaptive time step algorithm has 4.27 times the acceleration ratio compared with the fixed time step.

Table 3. The experimental result of 3D dam-break

Method	Total calculation time	Speedup ratio
The fixed time steps	128 min	
The adaptiving time steps	30 min	4.27

5 Conclusions

We propose a surface tension fluid simulation algorithm based on implicit incompressible SPH method. The experimental results show that our method has better surface tension effect compared to IISPH without surface tension. Under the influence of the surface tension, the fluid particles will gather with each other rather than disperse without restraint. At the same time, our method realizes the minimization of the fluid surface area, which makes the fluid surface become smoother. Furthermore, we suggested an adaptive time-stepping method which reduces the overall computation time for the simulation.

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