An Improved Anisotropic Kernels Surface Reconstruction Method for Multiphase Fluid

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Abstract. This paper improves the anisotropic kernels surface reconstruction method and apples it to multiphase immiscible fluid surface reconstruction. An unexpected phenomenon appears when using the anisotropic kernels surface reconstruction directly (e.g. the gap and overlap at the interface of multiphase fluid surface). We eliminate the gap by considering the neighbor particles of other phase fluid in the kernels function and eliminate the overlap by signed color field in the marching cube process. The improved method will be able to reconstruct a common surface at the interface of the multiphase fluid.

Keywords: SPH · Multiphase fluid · Surface reconstruction · Simulation visualization

1 Introduction

Because the surface representation of a fluid is crucial for realistic animation, methods for reconstructing and tracking fluid surfaces have been a topic of research since fluid simulation was first introduced in computer graphics [1–3].

The blobby sphere approach was introduced by Blinn [4], which defines a scalar field that is the sum of three-dimensional Guassian kernels [5] defined at a set of points. Surface are then taken to be smooth regions of the surface are then taken to be a particular iso-contour of the scalar field. Noting this problem, Zhu and Bridson [6] defined a new implicit surface model by averaging particle locations and their radii to enhance the surface smoothness of the blobby method. Adams et al. [3], further improved upon the method of Zhu and Bridson by tracking the particle-to-surface distances across simulation time steps. Williams [7] was the first to cast the problem of generating surfaces from particle data as a constrained of optimization problem.

Later, Yu and Turk [8] demonstrate very impressive results for the particle skinning problem, which used an anisotropic kernels and Laplacian smoothed particle positions to define a smoothed implicit surface [9]. But there will meet certain problem that gap and overlap will appear at the interface of multiphase fluid surface if we use it to multiphase surface reconstruction directly (see Fig. 1).

This paper improves the anisotropic kernels surface reconstruction method. We will first discuss the reasons of the gap and overlap, and then give the method to eliminate them. The contributions of our paper are as follow

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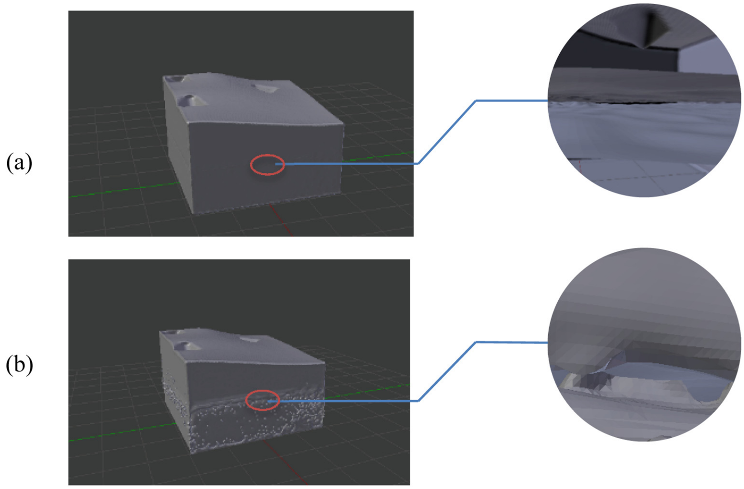


Fig. 1. Multiphase surface reconstruction. (a) using marching cube directly with overlap, (b) using anisotropic kernels directly with gap. (Color figure online)

- Consider the neighbor particles impact of other fluid in the Laplacian smoothing process and does not move the particles in the surface of multiphase fluid interface towards the inside.
- Combine the isotropic and anisotropic kernels function to eliminate the gap at the surface of multiphase fluid.
- We introduce the signed color field method to extract the surface in the marching cube process, that will eliminate the overlap.

2 SPH Framework

In SPH, fluid is discretized by particles carrying field quantities A_s. At any position r, these quantities can be evaluated by summing up the contributions of the neighboring particles j

$$A_s(\vec{r}) = \sum_j A_j \frac{m_j}{\rho_j} W(\vec{r} - \vec{r}_j, h) \tag{1}$$

where W is the smoothing kernel and h is the smoothing radius, m_j is the mass of particle, and ρ_j its density field. And then we can get the density field ρ_s

$$\rho_s(\vec{r}_i) = \sum_j \rho_j \frac{m_j}{\rho_j} W(\vec{r}_i - \vec{r}_j, h) = \sum_j m_j W(\vec{r}_i - \vec{r}_j, h) \tag{2}$$

The pressure p of particle is typically described as of the density of the fluid such as given by the Tait equation, which is

$$p_i = k\rho_0((\frac{\rho_i}{\rho_0})^{\gamma} - 1) \quad (3)$$

where k and γ are stiffness parameters and ρ_0 is the rest density of fluid. [10–12].

If we use the standard SPH framework to simulation multiphase fluid, a problem will arise. For particles close to the interface, the computed density is underestimated if they belong to the fluid with higher rest density, and overestimated otherwise. Solenthaler and Pajarola [13] present a formulation based on SPH which can handle this problem, and their main idea is to replace the density computation in SPH by a measure of particle densities and consequently derive new formulations for pressure and viscous forces.

They defined a particle density as

$$\delta_i = \sum_j W(\overrightarrow{r_{ij}}, h) \quad (4)$$

In this article, succeed to simulate multiple fluids with the method of Density Contrast which presented by B. Solenthaler and R. Pajarola.

3 Surface Reconstruction

3.1 Surface Field Computation

Yu and Turk made outstanding contributions to particle-based fluid surface re-modeling. Their approach used a single pass of Laplacian smoothing of particle positions, followed by defining a metaballs-like surface with anisotropic smoothing kernels. This method is perfect performed at single fluid surface reconstruction, but some gaps will arise at interface of multiphase fluid surface when used it to multiphase surface reconstruction of particle-based fluid.

We will save the particle position of each fluid when we used the method of Yu and Turk to reconstruct the multiphase fluid surface. Color field is computed by the particle position and density. This method is designed to capture the density more accurately by allowing the smoothing kernels to be anisotropic. We analysis of the causes of the gap should come from two aspects. On one hand, this method applies one step of diffusion smoothing to the location of kernel centers. The updated kernel centers are calculated by

$$\overline{x_i} = (1-\lambda)x_i + \lambda \sum_j w_{ij}x_j / \sum_j w_{xj} \quad (5)$$

where w is a suitable finite support weighting and λ is a constant with $0 < \lambda < 1$.

The function w_{ij} is an isotropic weighting function, and formulated as

$$w_{ij} = \begin{cases} 1 - ((\|x_i - x_j\|)/r_i)^3 & if \|x_i - x_j\| < r_i \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

We use λ between 0.9 and 1. According to the above two formulas, the particle position inside the fluid is relatively constant, but the particles at the surface will move towards inside. Obviously, we will not get the correct color field, because the computed fluid volume is smaller than the real color field. Shown in Fig. 2(b).

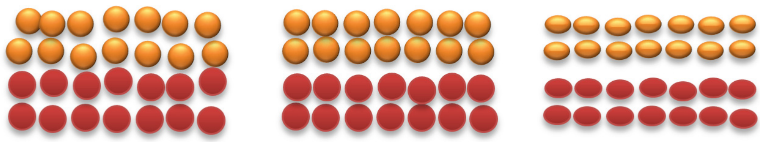


Fig. 2. (a) Particles real position, (b) After Laplacian smoothing, (c) After anisotropy (Color figure online)

On the other hand, this method used the anisotropic kernel in the surface reconstructing, and key of anisotropic kernel is to generate a spherical shape kernel in the fluid surface. The gap will be bigger at the interface of the multiphase fluid although the spherical shape kernel will make fluid surface more smoothing. Shown in Fig. 2(c).

We improved the anisotropic kernel method in multiphase surface reconstruction by considering the particle position of neighbor fluids. The improved $\overline{x_i}$ and w_{ikj} are calculated by

$$\overline{x_i} = (1-\lambda)x_i + \lambda \sum_{k}^{n} \sum_{j} w_{ikj} x_{kj} / \sum_{j}^{n} \sum_{j} w_{ikj} \tag{7}$$

$$w_{ikj} = \begin{cases} 1 - ((\|x_i - x_j\|)/r_i)^3 & if \|x_i - x_j\| < r_i \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where k is the fluid kind.

Because we have considered the neighbor particles of other fluids, the particle position in the interface of two fluids will not move towards inside. Additionally, the improved $\overline{x_i}$ and w_{ikj} can modify the anisotropic kernel function to isotropic at the interface of two fluids surface, and will not shrink fluid volume. The gap at the interface of two fluids surface will be eliminated.

3.2 Surface Extraction

Method of Marching Cube [14, 15] is usually adopted to extracting the mesh files after we computed the color field. For the reason that the different material of each fluid, we need to extract the mesh respectively. This will encounter a problem that is the overlap at the interface of the multiphase fluid. Shown in Fig. 1(a).

By analysis, the reason of overlap produce is that we used interpolation to compute the isometric surface in the marching cube. In the Fig. 3, We suppose the isometric value is 0 in mesh extracting process. we need to compute twice to get the two different

Fig. 3. Marching cube interpolation in two fluid (Color figure online)

fluid surface mesh files. In the first attempt, we use marching cube to extract the fluid which density is small (green solid). Because of the interpolation, the color field with value (green box) is in outside of the real surface (green solid). It is obvious that the extracting surface is near green box which is larger than the real. It is same when we extracting the fluid which density is large. Then we will get a large overlap at the interface of two fluids (green box and yellow box overlap).

Therefore, we propose signed color field to eliminate the overlap at the interface of the multiphase fluid. We define one fluid color field is positive and others fluid color field is negative to extract the one fluid surface mesh. successively, we extract all of the fluid surface mesh. The algorithm showing as follow (Table. 1).

Table 1. Algorithm of signed color field in the marching cube

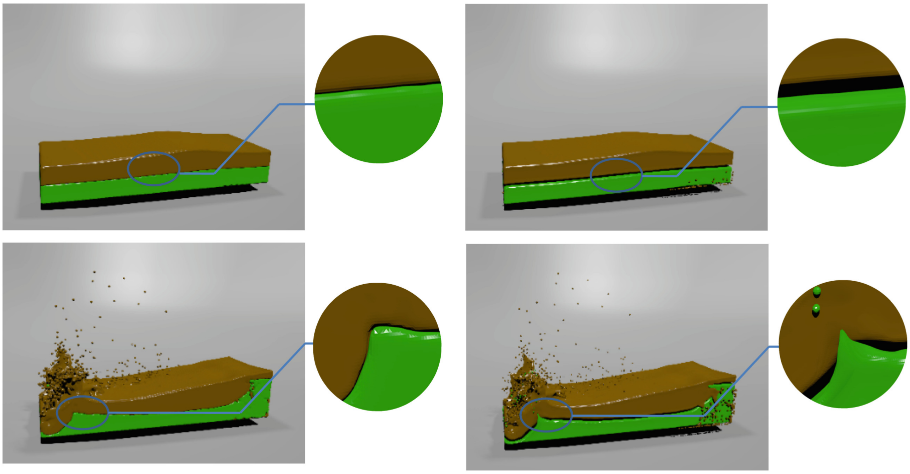
compute the fluid particle position
generate grid
for fluid i
Compute i *positive* color field
compute others *others* *positive* color field
interpolation $x = x_1 + (isovalue - \rho_1)\frac{(x_2 - x_1)}{(\rho_2 - \rho_1)}$
compute mesh file
save mesh file of i
merge mesh files

The signed color field method in the marching cube interpolation will always generate one surface at the interface of the multiphase fluid. This is because the value 0 from negative to positive is always unique and exists.

4 Results

In this section, we describe three simulations that are used to evaluate our surface reconstruction method: three fluids interface, double dam break simulation, two fluids mixing. Our experiment running platform is Intel (R) Xeon (R) CPU E5-2687 W v4 @3.00 GHz, 64-bit Windows operating system.

In the double dam break simulation, see Fig. 4. The green fluid density is 200 kg/m3, and the gray fluid density is 1000 kg/m3. Top render result is using anisotropic kernels method and marching cube directly, and bottom render result is using our improved method. The surface reconstruction time spent shown as Table 1. Both the result is smoothing and spend time is Almost the same, but our method the gap at the interface of two fluids is thinner.

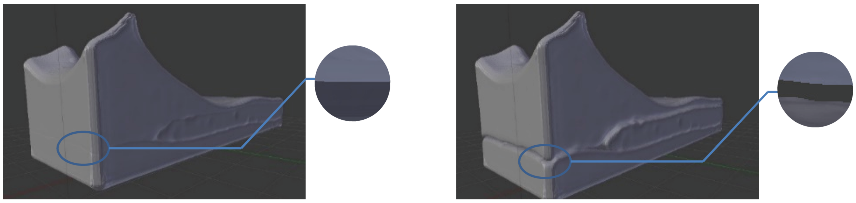


(a) (b)

Fig. 4. Double dam break simulation, (a) our improved method, (b) using anisotropic kernels method and marching cube directly. (Color figure online)

Figure 5 shows three fluids mixing surface reconstruction results. (a) using our improved method. (b) using anisotropic kernels method and marching cube directly. Note that our result has narrow gap at the interface of multiphase surface, and without overlap in the inner interface of two fluid surface.

Figure 6 shows a frame of double dam break simulation, which rendered by the Blender (Table 2).



(a) (b)

Fig. 5. Three immiscible fluid mixing, (a) our improved method, (b) using anisotropic kernels method and marching cube directly.

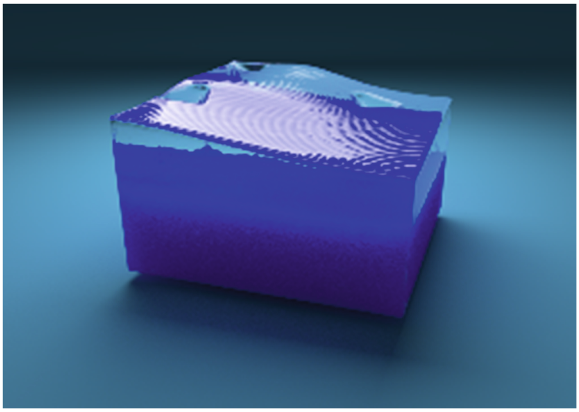


Fig. 6. Double dam break simulation, rendered by the Blender.

Table 2. Surface reconstruction time spent

Method	Total frame	Total time
Anisotropic kernels	600	19.091 h
Our method	600	20.197 h

5 Conclusion

We improve the anisotropic kernels surface reconstruction method then apply it to multiphase immiscible fluid. Using this method to reconstruct the surface of multiphase immiscible fluid will get smoothing surface and without gap or overlap problem. Additionally, it is easy to implement and does not require more time than the original method.

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